The Trigonometric Parallax
and
Proper Motion of Barnard's Star

Error and Precision in Small-Telescope Astrometry

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Error and Precision in Small-Telescope Astrometry

- Introduction
- Barnard's Star
- Astrometry Workflow
- Sources of Error
- Extrinsic
  - Optics
  - Image Capture
  - Image Measurement
  - Reference Star Properties
  - Astrometric Solution
- Results for Barnard's Star

Introduction

- History
- Instrumentation
- Observations
- Accuracy and Precision

History

- Began as a student project question…
  - Is it possible to detect the proper motion of Barnard's Star in a three-night workshop?
- Answer: Possible in a single night!
- Analysis of data taken 2009-2010
- On-going project…

Instrumentation

- Alpaca Meadows Observatory (-122.6,+44.8)
  - 8-inch $f/4$ Newtonian (Vixen R200SS)
  - TeleVue ParaCorr coma corrector
  - QSI 532ws CCD camera
  - Schüler B, V, Rc, Ic, clear filters
- Pine Mountain Observatory (-120.9,+43.8)
  - Celestron 11-inch $f/10$ EdgeHD
  - Same CCD camera and filters

Observations

- 20 nights
  - 11 nights in 2009, experimenting
  - 9 nights in 2010, focused on data collection
- 836 useable images
  - 2009: BVRI, VR, and V filters, 10 to 60 sec
    - Between 14 and 92 images per night
  - 2010: V filter only, 20, 40, and 60 sec
    - Usually 60 images per night

Accuracy & Precision

Meas = True + Syst $\pm$ Rand

We call Meas "accurate" when it satisfies:

Syst << Meas and Syst < Rand and Meas $\cong$ True

We call Meas "precise" when it satisfies:

Rand << Meas and Rand < Syst

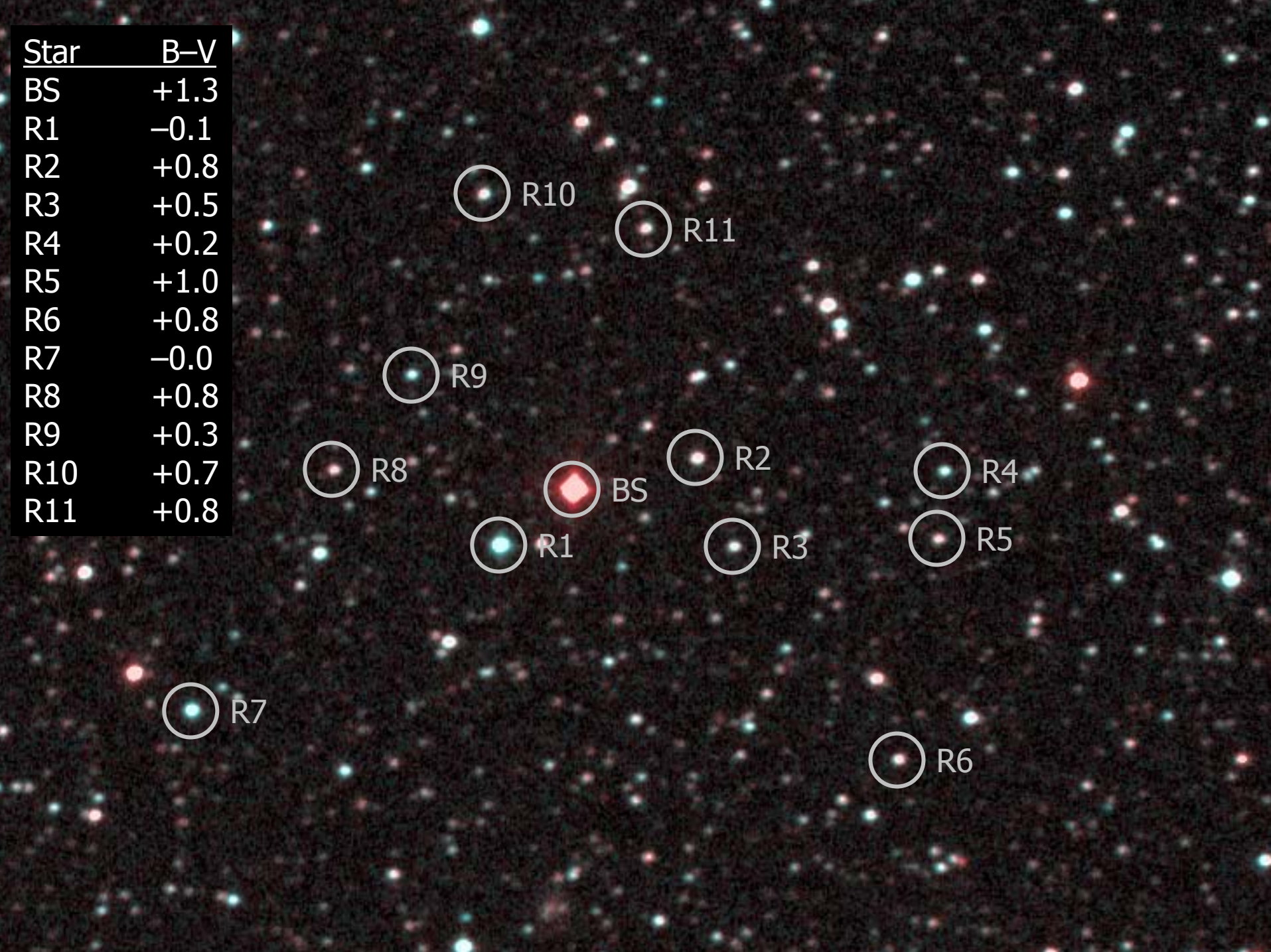
True = true value
Meas = measured value
Syst = systematic error, or bias
Rand = random error, or uncertainty

Barnard's Star

- Star Field
- Properties
- Proper Motion
- Trigonometric Parallax

Barnard's Star Field in V

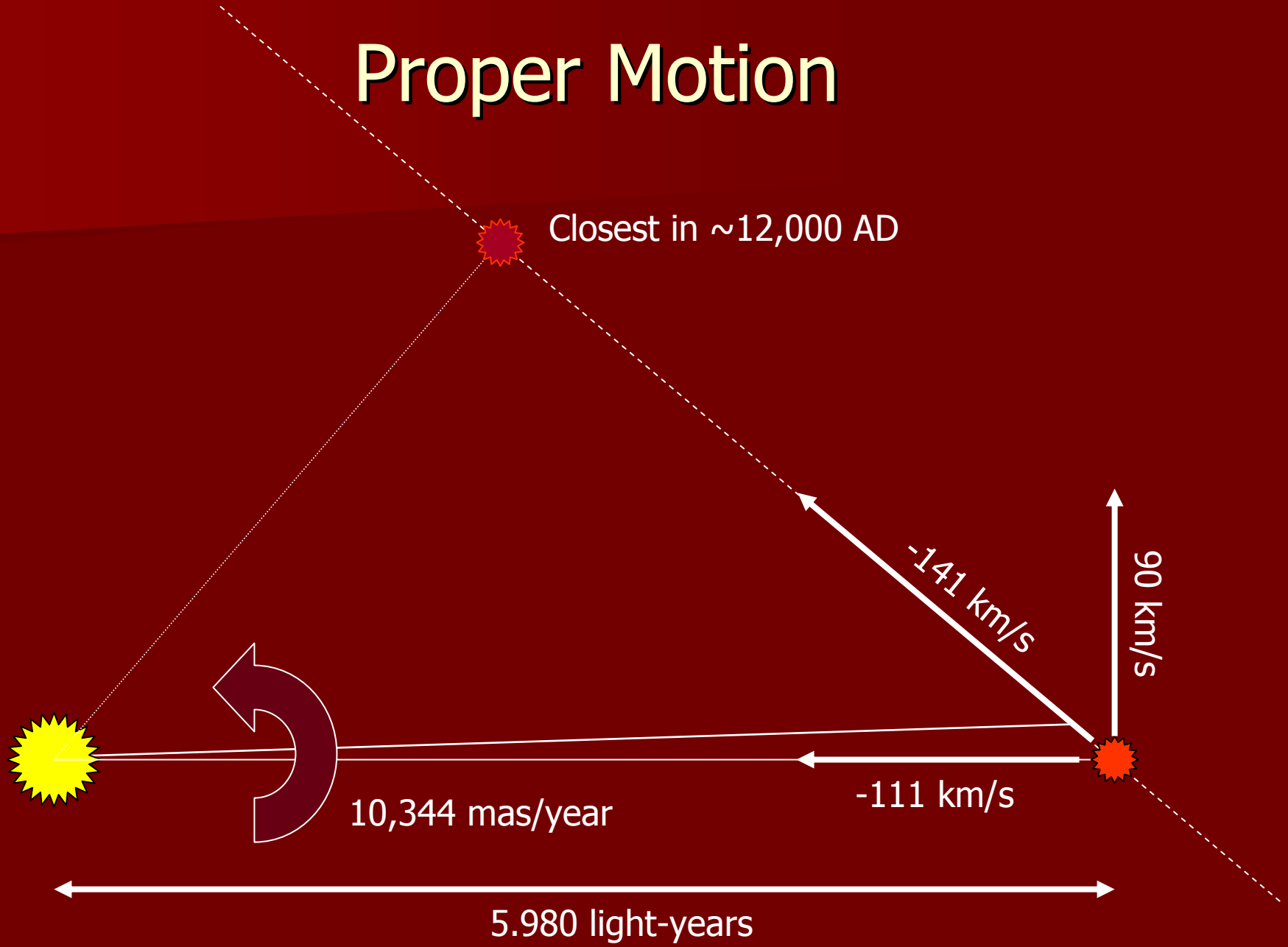
Barnard's Star Field (BVR color)


Star B–V
BS +1.3
R1 –0.1
R2 +0.8
R3 +0.5
R4 +0.2
R5 +1.0
R6 +0.8
R7 –0.0
R8 +0.8
R9 +0.3
R10 +0.7
R11 +0.8
R10
R11
R9
R8
BS
R2
R4
R1
R3
R5
R7
R6

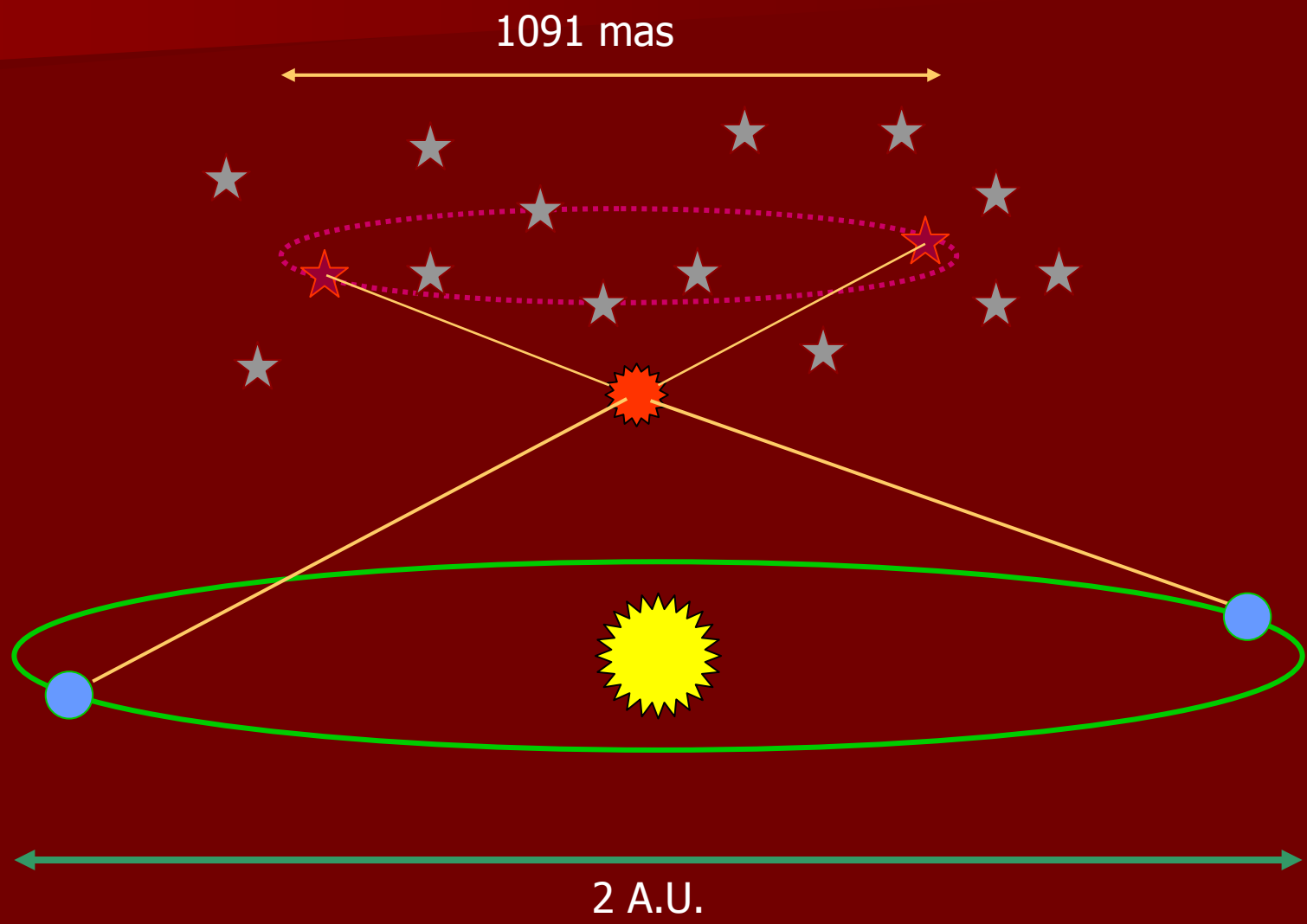
Properties

- Barnard's Star = BD+4º3561 = TYC 425-2502
- Location: Ophiuchus
- Coordinates: $17^h57^m48.5$ +4º41'36" (J2000)
- Apparent Magnitude: V = 9.54 (variable)
- Spectral Class: M4V (red dwarf)
- T_{eff} = 3100±100
- Proper Motion: 10.33777 arc-seconds/year
- Parallax: 0.5454 arc-seconds
- Distance: 5.980±0.003 light-years
- Radial Velocity: –110.6 km/second
- Rotation Period: 130.4 days

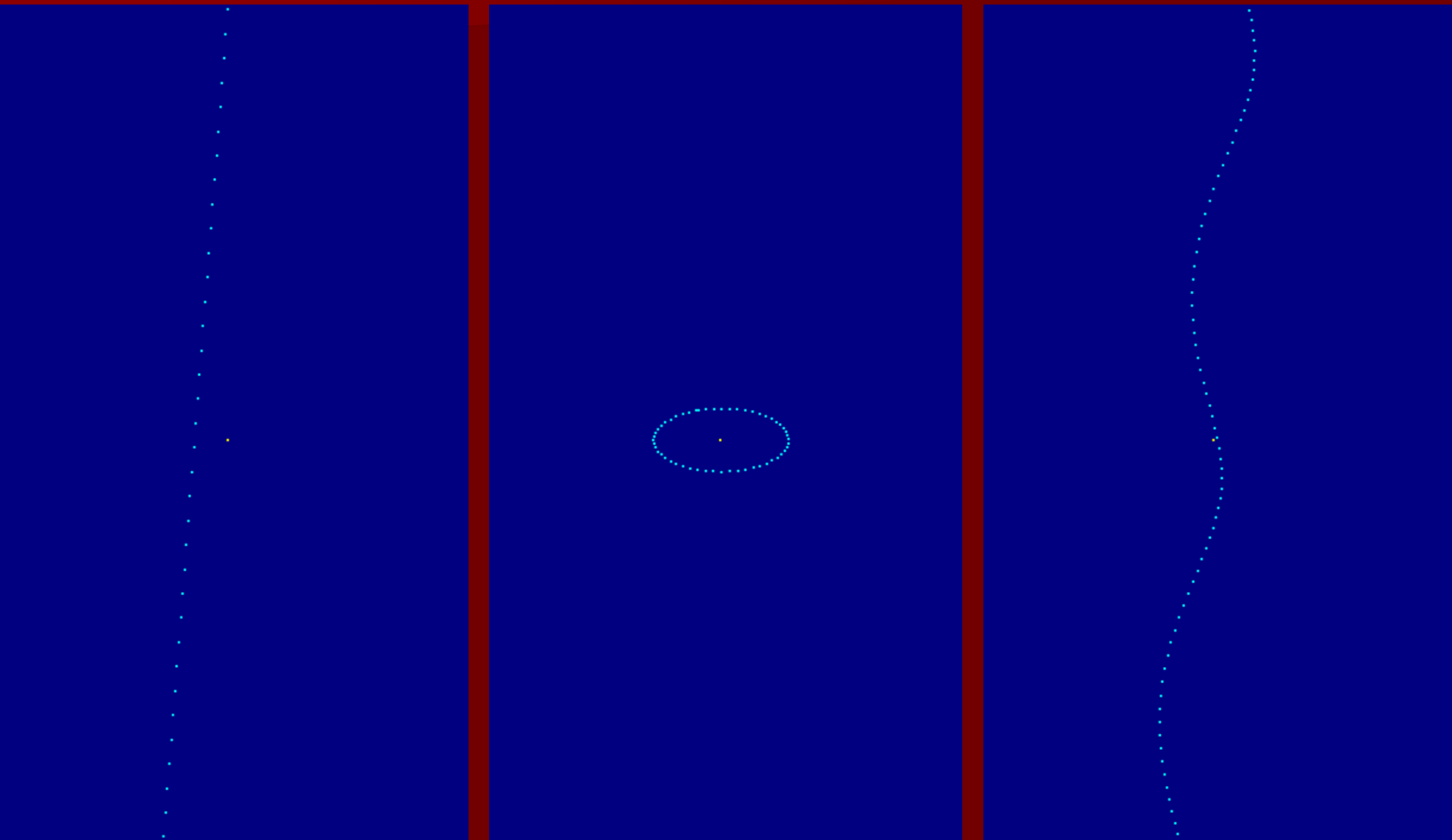
Proper Motion



Trigonometric Parallax



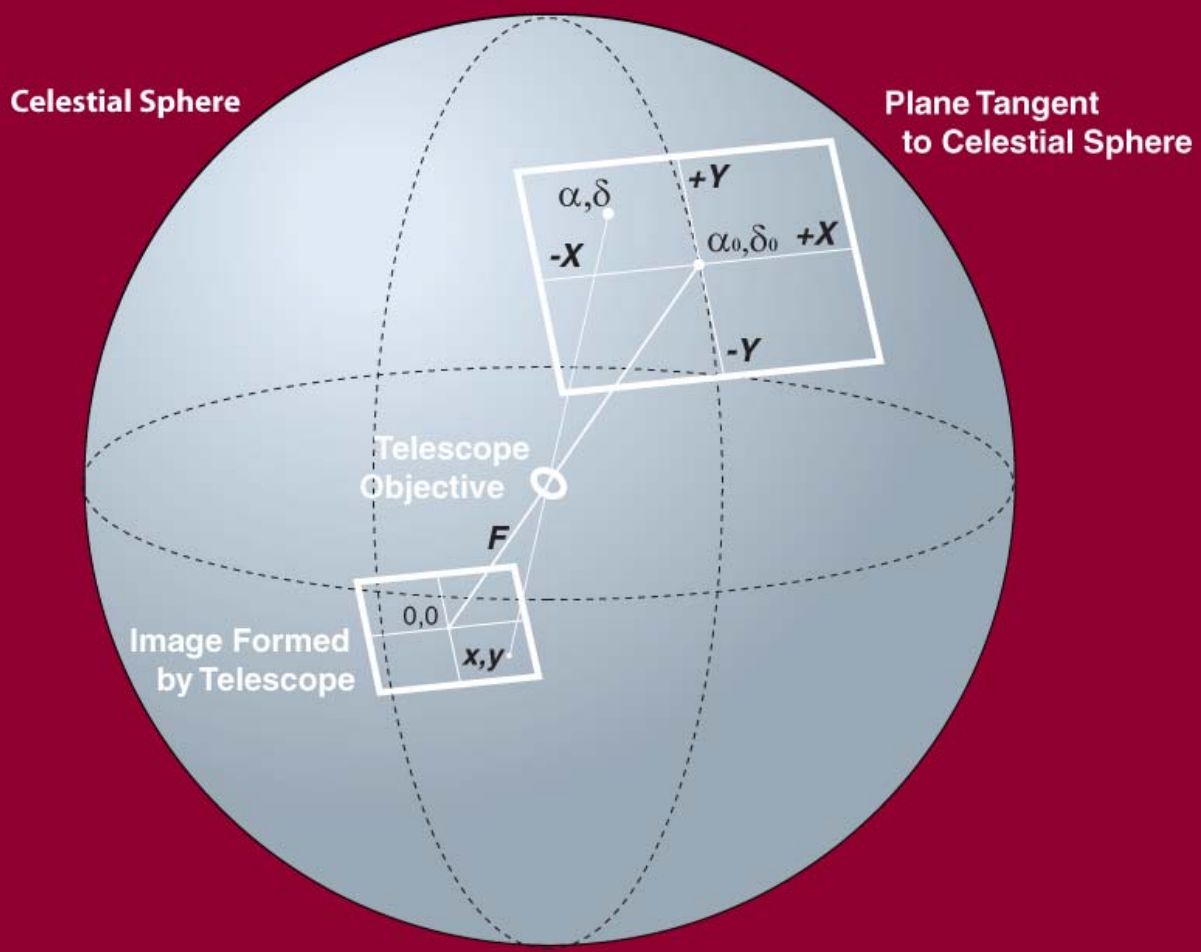
Proper Motion, Parallax, and the Path of Barnard's Star



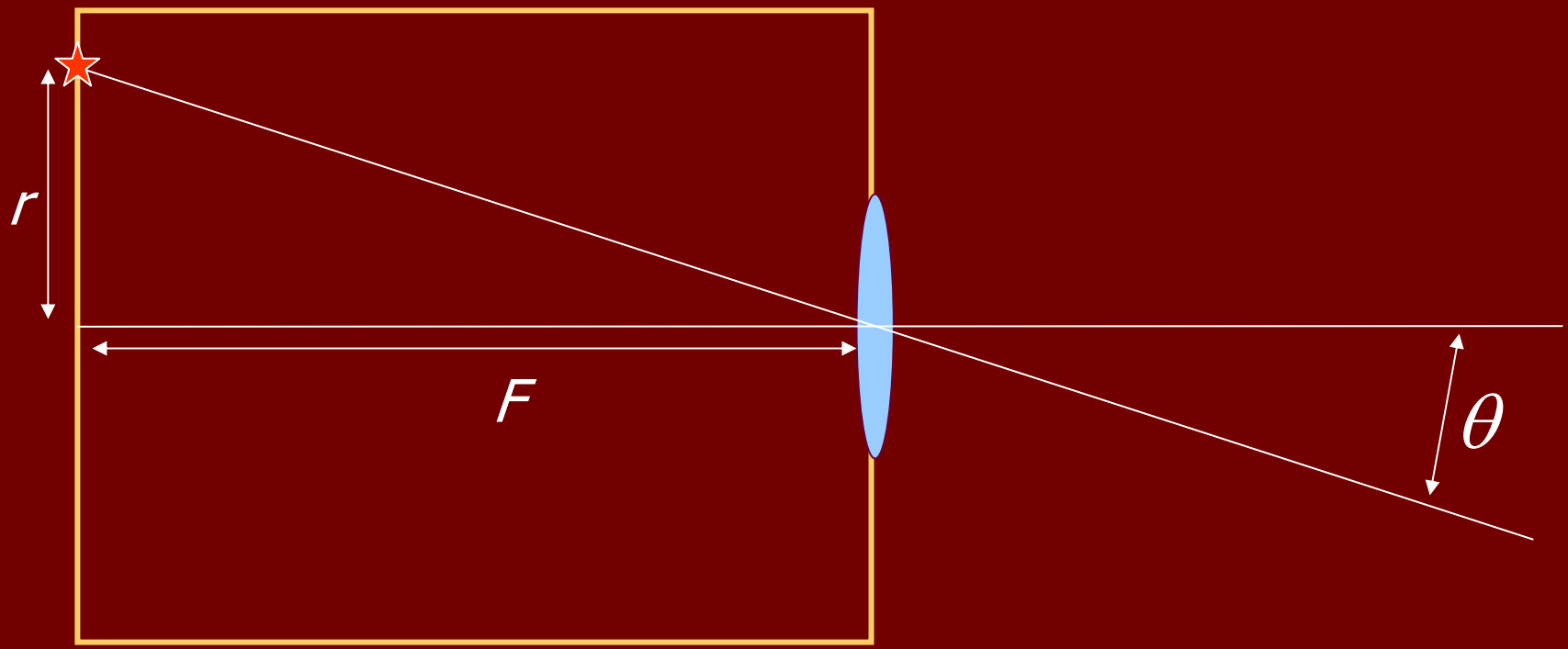
Astrometry Workflow

- Optical System
- Image Capture
- Image Measurement
- Reference Star Data
- Astrometric Solution

Idealized Projection

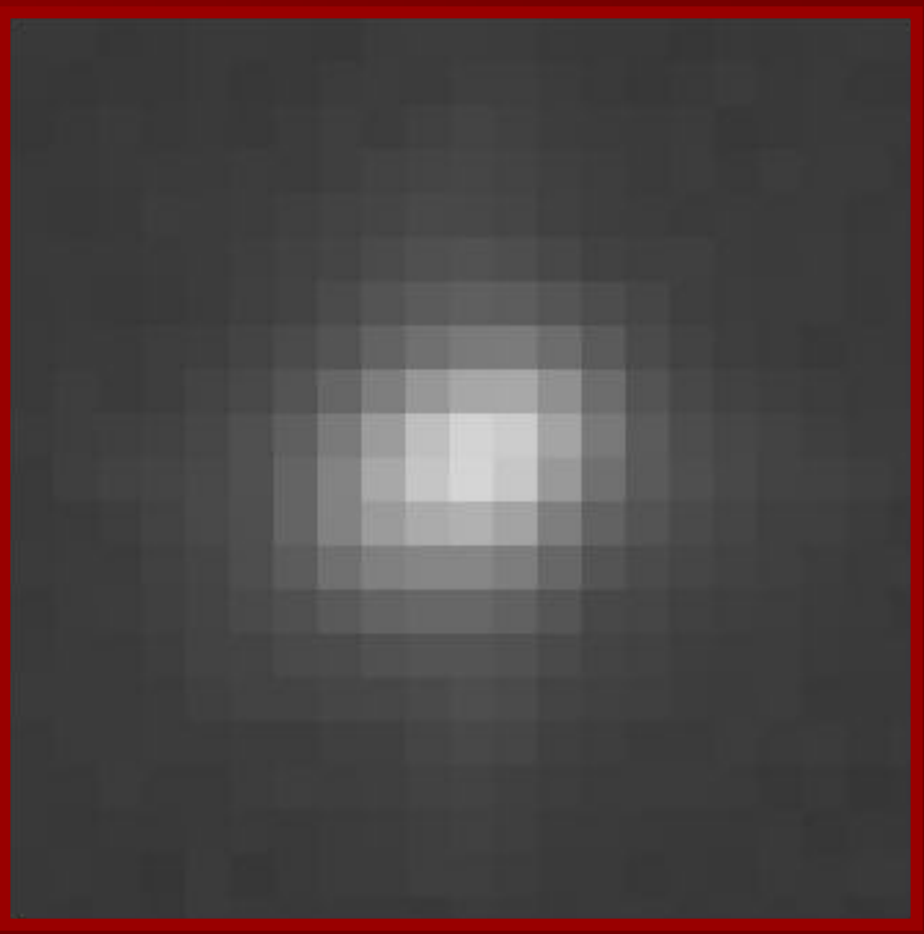


Idealized Telescope



$r = F\tan(\theta)$

Image Capture / Image Measurement



- How accurately can we determine the (x,y) location of the star?
- What is the relationship between location accuracy and brightness?

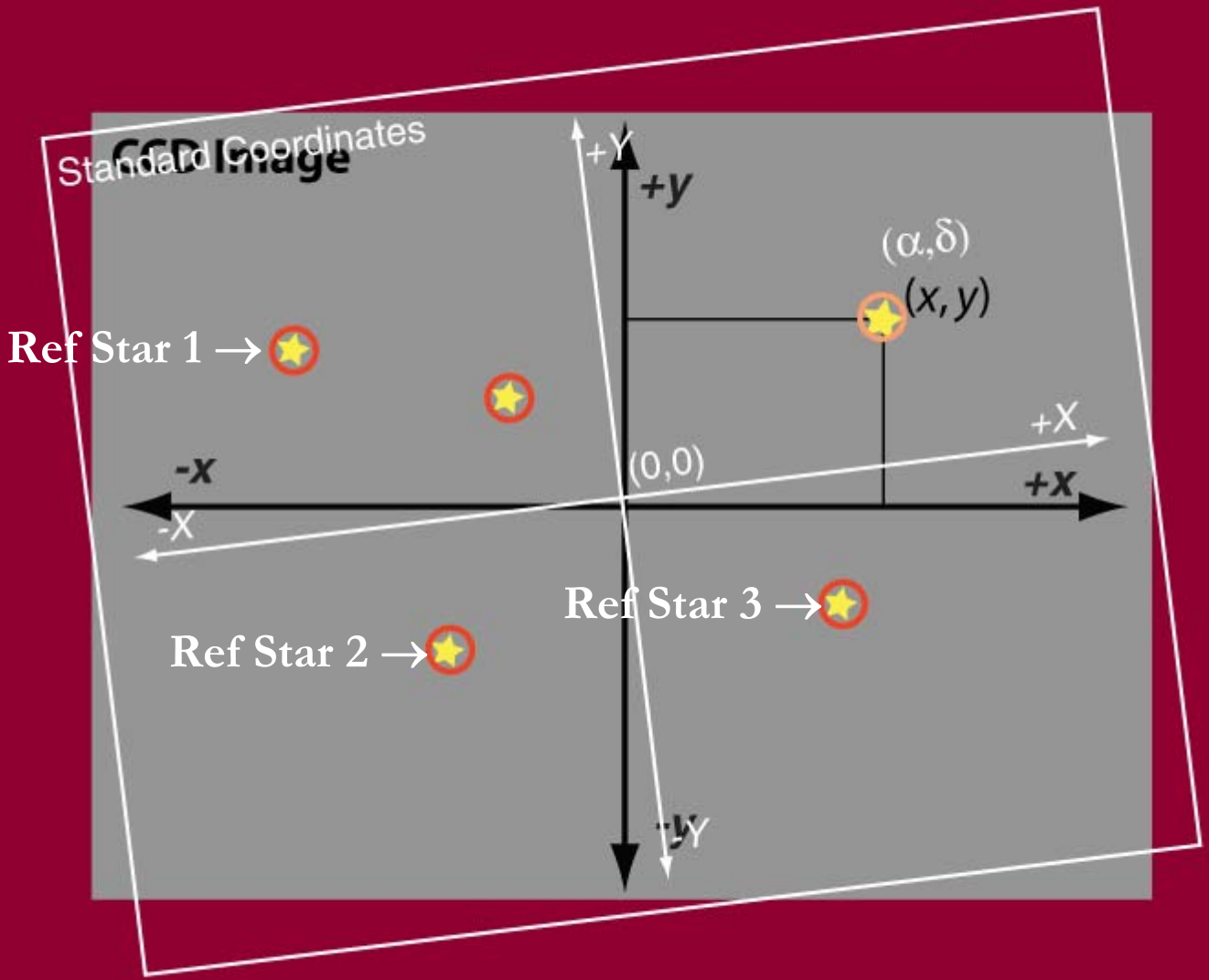
Reference Stars: $(\alpha,\delta) \rightarrow (X,Y)$

- When you shoot an image, you're mapping the celestial spherical onto a plane surface.
- This occurs for all the stars in the image, both the target stars and the reference stars.
- The standard (X,Y) coordinates of a star at (α,δ) for an image centered on (α_0,δ_0) are:

$$X = (\cos\delta \sin(\alpha-\alpha_0))/d$$

$$Y = (\sin\delta_0 \cos\delta \cos(\alpha-\alpha_0) - \cos\delta_0 \sin\delta)/d$$

where $d = \cos\delta_0 \cos\delta \cos(\alpha-\alpha_0)+\sin\delta_0 \sin\delta$.



By offsetting, rotating, and scaling standard coordinates, we can link each reference star with its counterpart in the image.

$(x,y) \rightarrow (X,Y)$

- To offset, rotate, and scale coordinates:
  - $X = x \cos\rho / F + y \sin\rho / F + x_{offset} / F$
  - $Y = x \sin\rho / F + y \cos\rho / F + y_{offset} / F$
- But we do not know ρ, F, or the offsets.
- However, for each reference star, we know:
  - (X,Y) standard coordinates, and
  - (x,y) image coordinates.

Linking the Coordinates

- Suppose we have three reference stars.
- For each star, we know (x,y) and (X,Y).
  - $X_1 = ax_1 + by_1 + c$ and $Y_1 = dx_1 + dy_1 + f$
  - $X_2 = ax_2 + by_2 + c$ and $Y_2 = dx_2 + dy_2 + f$
  - $X_3 = ax_3 + by_3 + c$ and $Y_3 = dx_3 + dy_3 + f$.
- Three equations, three unknowns $\rightarrow$ solvable.
- -
- Suppose we have *many* reference stars.
- Solve by the method of least squares: errors are $\varepsilon_{x,n}$ and $\varepsilon_{y,n}$.
  - $\varepsilon_{x,n} = ax_n + by_n + c - X_n$ and $\varepsilon_{y,n} = dx_n + dy_n + f - Y_n$
- The residual, σ, expresses the error in the fit.

Computing Target Coordinates

- From reference stars, we find *a, b, c, d, e,* and *f*.
- The standard coordinates of the target are:
  - $X_{target} = ax_{target} + by_{target} + c$, and
  - $Y_{target} = dx_{target} + ey_{target} + f$
- Given (X, Y) for the target, it's (α,δ) is:
  - $\delta = \arcsin((\sin\delta_0 + Y\cos\delta_0)/(\sqrt{1+X^2+Y^2}))$, and
  - $\alpha = \alpha_0 + \arctan(X/(\cos\delta_0 + Y\sin\delta_0))$.
- Results are expressed with the residual:
  - $\alpha \pm \sigma_\alpha$
  - $\delta \pm \sigma_\delta$

Astrometric Data from Each Set of Images

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	AIP4Win v2.3.33 Magnitude Measurement Tool												
2	Astrometric Coordinates in Text Format												
3	Reference stars = 11												
4	Now = 10/8/2010 9:48:31 PM												
5	Seq#	Julian Day	Focal[mm]	PA[d.d]	X[pix]	Y[pix]	RA[d.d]	Dec[d.d]	RArms	DCrms	HH MM SS.ss	+DD MM SS.s	FileName
6	0	2455465.4719	911.0738	174.8909	1109.1150	724.255	269.449540	4.724217	0.145	0.092	17 57 47.89	+04 43 27.2	Barnstar-001-V60s.fit
7	1	2455465.4729	911.1903	174.9034	1110.3390	725.270	269.449559	4.724245	0.137	0.086	17 57 47.89	+04 43 27.3	Barnstar-002-V60s.fit
8	2	2455465.4738	911.0766	174.8860	1110.7570	725.883	269.449519	4.724284	0.118	0.048	17 57 47.88	+04 43 27.4	Barnstar-003-V60s.fit
9	3	2455465.4748	911.1803	174.8925	1111.6590	726.666	269.449526	4.724268	0.160	0.072	17 57 47.89	+04 43 27.4	Barnstar-004-V60s.fit
10	4	2455465.4758	911.2407	174.8792	1112.4750	726.338	269.449534	4.724249	0.101	0.083	17 57 47.89	+04 43 27.3	Barnstar-005-V60s.fit
11	5	2455465.4767	910.8706	174.8941	1113.0990	726.950	269.449537	4.724278	0.149	0.112	17 57 47.89	+04 43 27.4	Barnstar-006-V60s.fit
12	6	2455465.4777	910.8468	174.8869	1113.2640	726.589	269.449533	4.724270	0.144	0.086	17 57 47.89	+04 43 27.4	Barnstar-007-V60s.fit
13	7	2455465.4787	910.9625	174.8899	1114.2280	726.405	269.449547	4.724234	0.108	0.100	17 57 47.89	+04 43 27.2	Barnstar-008-V60s.fit
14	8	2455465.4796	920.5098	173.9535	1114.7880	726.566	269.449874	4.724502	4.913	3.134	17 57 47.97	+04 43 28.2	Barnstar-009-V60s.fit
15	9	2455465.4806	911.0141	174.8864	1115.2200	726.386	269.449559	4.724228	0.077	0.081	17 57 47.89	+04 43 27.2	Barnstar-010-V60s.fit
16	10	2455465.4816	911.1652	174.8776	1115.4930	726.321	269.449523	4.724258	0.154	0.109	17 57 47.89	+04 43 27.3	Barnstar-011-V60s.fit
17	11	2455465.4825	911.1594	174.8983	1115.6550	726.408	269.449525	4.724241	0.135	0.082	17 57 47.89	+04 43 27.3	Barnstar-012-V60s.fit
18	12	2455465.4835	910.9489	174.8854	1115.9300	726.182	269.449530	4.724245	0.108	0.112	17 57 47.89	+04 43 27.3	Barnstar-013-V60s.fit
19	13	2455465.4845	910.8863	174.8830	1116.3670	726.080	269.449521	4.724284	0.119	0.098	17 57 47.89	+04 43 27.4	Barnstar-014-V60s.fit
20	14	2455465.4855	910.8912	174.8717	1116.3460	725.982	269.449526	4.724279	0.160	0.087	17 57 47.89	+04 43 27.4	Barnstar-015-V60s.fit
21	15	2455465.4864	911.0373	174.8802	1116.6080	726.036	269.449510	4.724277	0.099	0.109	17 57 47.88	+04 43 27.4	Barnstar-016-V60s.fit
22	16	2455465.4874	911.0205	174.8933	1116.8730	725.895	269.449500	4.724265	0.143	0.093	17 57 47.88	+04 43 27.4	Barnstar-017-V60s.fit
23	17	2455465.4884	911.1496	174.8793	1117.4670	725.950	269.449506	4.724290	0.134	0.025	17 57 47.88	+04 43 27.4	Barnstar-018-V60s.fit
24	18	2455465.4893	910.9007	174.8687	1117.2820	726.238	269.449525	4.724272	0.152	0.100	17 57 47.89	+04 43 27.4	Barnstar-019-V60s.fit
25	19	2455465.4903	911.0019	174.8688	1117.9900	726.291	269.449579	4.724262	0.130	0.037	17 57 47.90	+04 43 27.3	Barnstar-020-V60s.fit
26	20	2455465.4913	911.3158	174.8904	1117.8650	726.206	269.449553	4.724259	0.138	0.091	17 57 47.89	+04 43 27.3	Barnstar-021-V60s.fit
27	21	2455465.4922	910.8112	174.8790	1118.2040	726.087	269.449518	4.724286	0.152	0.093	17 57 47.88	+04 43 27.4	Barnstar-022-V60s.fit
28	22	2455465.4932	910.9561	174.8623	1118.4220	725.857	269.449516	4.724302	0.115	0.085	17 57 47.88	+04 43 27.5	Barnstar-023-V60s.fit
29	23	2455465.4942	911.1515	174.8843	1118.6280	725.246	269.449493	4.724287	0.151	0.097	17 57 47.88	+04 43 27.4	Barnstar-024-V60s.fit
30	24	2455465.4951	910.7821	174.8772	1119.1060	725.354	269.449541	4.724247	0.136	0.089	17 57 47.89	+04 43 27.3	Barnstar-025-V60s.fit
31	25	2455465.4961	910.8367	174.8403	1119.3980	725.095	269.449498	4.724294	0.142	0.089	17 57 47.88	+04 43 27.5	Barnstar-026-V60s.fit
32	26	2455465.4971	910.9794	174.8579	1119.2390	725.054	269.449512	4.724298	0.132	0.091	17 57 47.88	+04 43 27.5	Barnstar-027-V60s.fit
33	27	2455465.4980	911.0343	174.8766	1119.6120	724.681	269.449508	4.724280	0.100	0.076	17 57 47.88	+04 43 27.4	Barnstar-028-V60s.fit
34	28	2455465.4990	911.1055	174.9062	1120.0520	724.224	269.449503	4.724268	0.144	0.070	17 57 47.88	+04 43 27.4	Barnstar-029-V60s.fit
35	29	2455465.5000	911.0400	174.8752	1120.6640	724.107	269.449534	4.724269	0.141	0.114	17 57 47.89	+04 43 27.4	Barnstar-030-V60s.fit
36	30	2455465.5009	911.1117	174.8649	1120.3910	724.219	269.449534	4.724270	0.151	0.071	17 57 47.89	+04 43 27.4	Barnstar-031-V60s.fit
37	31	2455465.5019	910.9768	174.8785	1121.4030	724.389	269.449531	4.724254	0.094	0.121	17 57 47.89	+04 43 27.3	Barnstar-032-V60s.fit
38	32	2455465.5029	910.8195	174.8627	1121.0790	724.098	269.449534	4.724268	0.173	0.134	17 57 47.89	+04 43 27.4	Barnstar-033-V60s.fit
39	33	2455465.5038	910.9042	174.8599	1121.6070	723.931	269.449509	4.724293	0.136	0.089	17 57 47.88	+04 43 27.5	Barnstar-034-V60s.fit
40	34	2455465.5048	911.0821	174.8755	1121.8740	723.437	269.449559	4.724231	0.117	0.088	17 57 47.89	+04 43 27.2	Barnstar-035-V60s.fit

Sources of Error

- Extrinsic
- Optics
- Image Capture
- Image Measurement
- Reference Star Properties
- Astrometric Solution

Astrometry:
Potential Sources of Error 1

- Extrinsic
  - Proper motion of reference stars (displaced stars)
  - Atmospheric refraction (displaced stars)
- Optical System
  - Failure of $r = F \cdot \tan(\theta)$ mapping (displaced images)
  - Lateral chromatic aberration (displaced images)
- Image Capture
  - Poor focus (centroid error)
  - Bad tracking (centroiding asymmetric images)
  - Calibration (images displaced by hot pixels)
  - Time error (HA and zenith-distance errors)

Astrometry:
Potential Sources of Error 2

- Target and Ref Star Measurement
  - Faint background stars (centroid errors)
  - Centroid extraction (incorrect centroid)
- Reference Star Properties
  - Position errors (skewed astrometric solution)
  - Proper motion errors (skewed astrometric solution)
  - Dispersion errors (skewed astrometric solution)
- Astrometric Solution
  - Reference star proper motions
  - Differential refraction errors
  - Incorrect coding of solution algorithm

Differential Atmospheric Refraction

- Atmospheric Refraction
  - $R = (n\text{-}1) \tan z$
    - R is the angle of refraction
    - n is the refractive index of air
    - z is the zenith distance
- Differential Atmospheric Refraction
  - $\Delta R = (n(\lambda) - n(\lambda_0)) \tan z$
  - Blue more strongly refracted than red
- Effective Wavelength of Observation
  - Slope of spectrum across bandwidth of filter
  - Barnard's Star displaced relative to bluer stars

Differential Atmospheric Dispersion

Atmospheric Parameters

20.0	Temperature [Celsius]
40	Relative Humidity as %
1000	Pressure [mbar]

Wavelength Range

350	Lambda Min [nm]
850	Lambda Max [nm]
50	Lambda Step [nm]

Airmass Range

1.3	Airmass Minimum
4.0	Airmass Maximum
0.100	Airmass Step

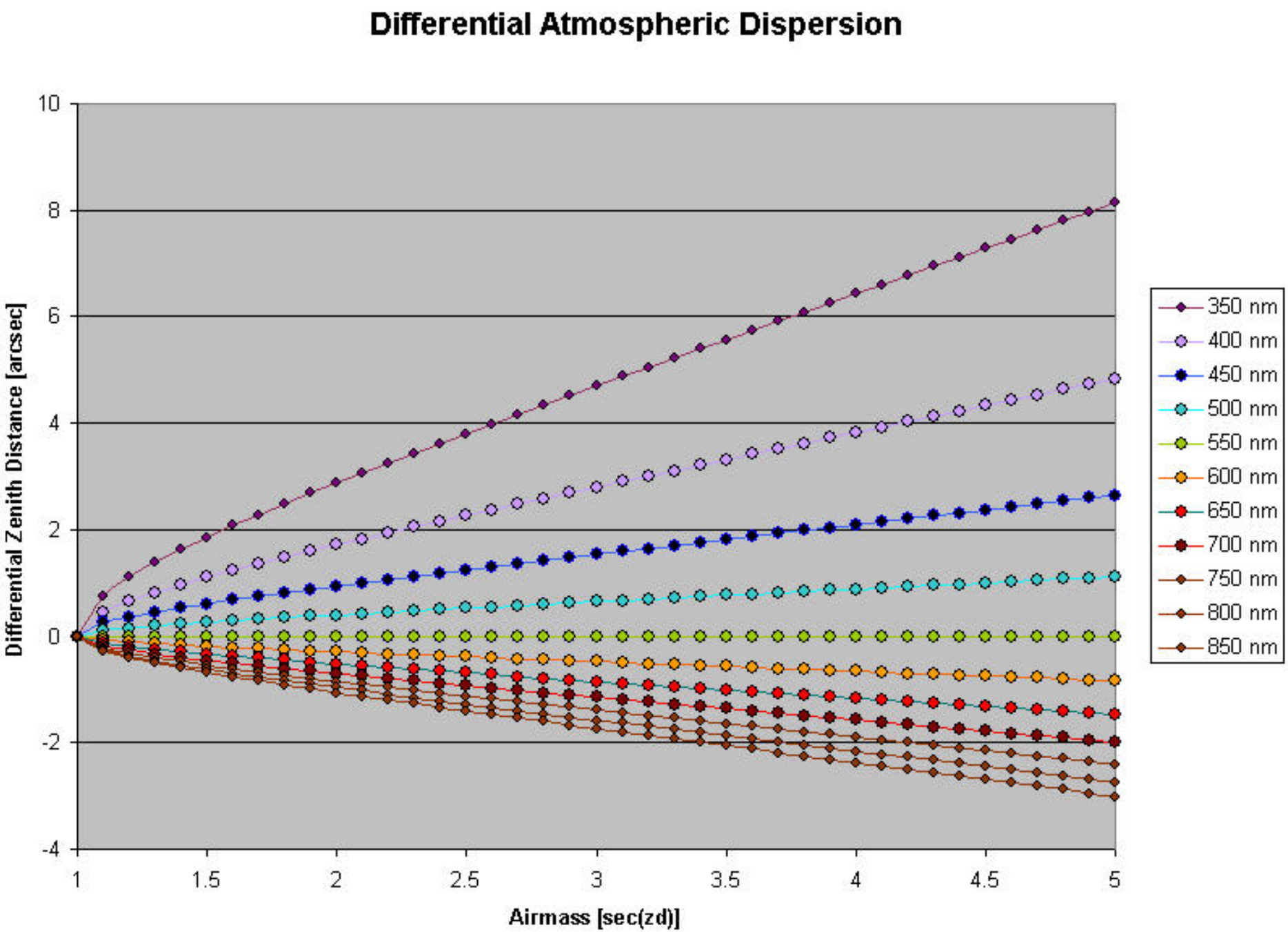
Reference Wavelength

550 Lambda Ref [nm]

Compute | Defaults | Save To File

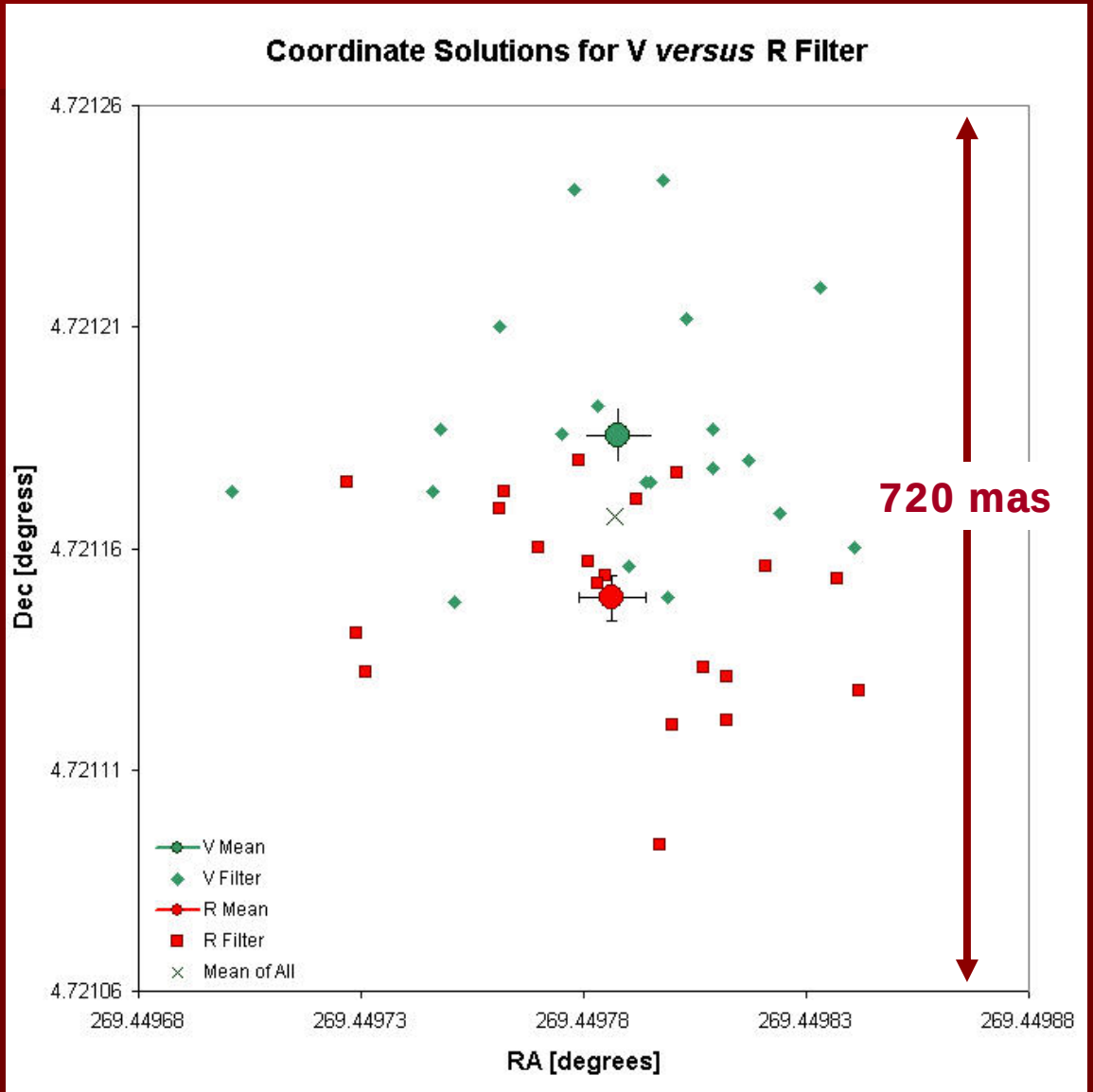
SecZD	ZD(deg)	350	400	450	500	550	600	650	700	750	800	850
1.300	39.7	+1.379	+0.820	+0.449	+0.189	+0.000	-0.142	-0.252	-0.338	-0.408	-0.464	-0.511
1.400	44.4	+1.627	+0.967	+0.529	+0.223	+0.000	-0.168	-0.297	-0.399	-0.481	-0.548	-0.603
1.500	48.2	+1.857	+1.104	+0.604	+0.255	+0.000	-0.191	-0.339	-0.455	-0.549	-0.625	-0.688
1.600	51.3	+2.074	+1.233	+0.675	+0.284	+0.000	-0.214	-0.379	-0.509	-0.613	-0.698	-0.768
1.700	54.0	+2.283	+1.357	+0.743	+0.313	+0.000	-0.235	-0.417	-0.560	-0.675	-0.769	-0.846
1.800	56.3	+2.485	+1.477	+0.809	+0.341	+0.000	-0.256	-0.454	-0.610	-0.735	-0.837	-0.921
1.900	58.2	+2.683	+1.595	+0.873	+0.368	+0.000	-0.277	-0.490	-0.658	-0.793	-0.903	-0.994
2.000	60.0	+2.876	+1.710	+0.936	+0.394	+0.000	-0.297	-0.525	-0.706	-0.850	-0.968	-1.066
2.100	61.6	+3.066	+1.823	+0.998	+0.420	+0.000	-0.316	-0.560	-0.752	-0.907	-1.032	-1.136
2.200	63.0	+3.254	+1.934	+1.059	+0.446	+0.000	-0.336	-0.594	-0.798	-0.962	-1.095	-1.206
2.300	64.2	+3.439	+2.045	+1.119	+0.472	+0.000	-0.355	-0.628	-0.844	-1.017	-1.158	-1.274
2.400	65.4	+3.623	+2.154	+1.179	+0.497	+0.000	-0.374	-0.662	-0.889	-1.071	-1.220	-1.342
2.500	66.4	+3.805	+2.262	+1.238	+0.522	+0.000	-0.392	-0.695	-0.933	-1.125	-1.281	-1.410
2.600	67.4	+3.985	+2.369	+1.297	+0.546	+0.000	-0.411	-0.728	-0.978	-1.178	-1.342	-1.477
2.700	68.3	+4.165	+2.476	+1.355	+0.571	+0.000	-0.429	-0.761	-1.022	-1.231	-1.402	-1.543
2.800	69.1	+4.343	+2.582	+1.413	+0.596	+0.000	-0.448	-0.793	-1.065	-1.284	-1.462	-1.609
2.900	69.8	+4.520	+2.687	+1.471	+0.620	+0.000	-0.466	-0.826	-1.109	-1.336	-1.522	-1.675
3.000	70.5	+4.697	+2.792	+1.528	+0.644	+0.000	-0.484	-0.858	-1.152	-1.389	-1.581	-1.740
3.100	71.2	+4.873	+2.896	+1.585	+0.668	+0.000	-0.502	-0.890	-1.195	-1.441	-1.640	-1.805
3.200	71.8	+5.048	+3.001	+1.642	+0.692	+0.000	-0.520	-0.922	-1.238	-1.492	-1.699	-1.870
3.300	72.4	+5.222	+3.104	+1.699	+0.716	+0.000	-0.538	-0.954	-1.281	-1.544	-1.758	-1.935
3.400	72.9	+5.396	+3.208	+1.756	+0.740	+0.000	-0.556	-0.986	-1.324	-1.595	-1.817	-1.999
3.500	73.4	+5.570	+3.311	+1.812	+0.764	+0.000	-0.574	-1.017	-1.366	-1.647	-1.875	-2.064
3.600	73.9	+5.743	+3.414	+1.869	+0.787	+0.000	-0.592	-1.049	-1.409	-1.698	-1.933	-2.128
3.700	74.3	+5.915	+3.516	+1.925	+0.811	+0.000	-0.610	-1.080	-1.451	-1.749	-1.991	-2.192
3.800	74.7	+6.088	+3.619	+1.981	+0.835	+0.000	-0.628	-1.112	-1.494	-1.800	-2.049	-2.255
3.900	75.1	+6.260	+3.721	+2.037	+0.858	+0.000	-0.645	-1.143	-1.536	-1.851	-2.107	-2.319

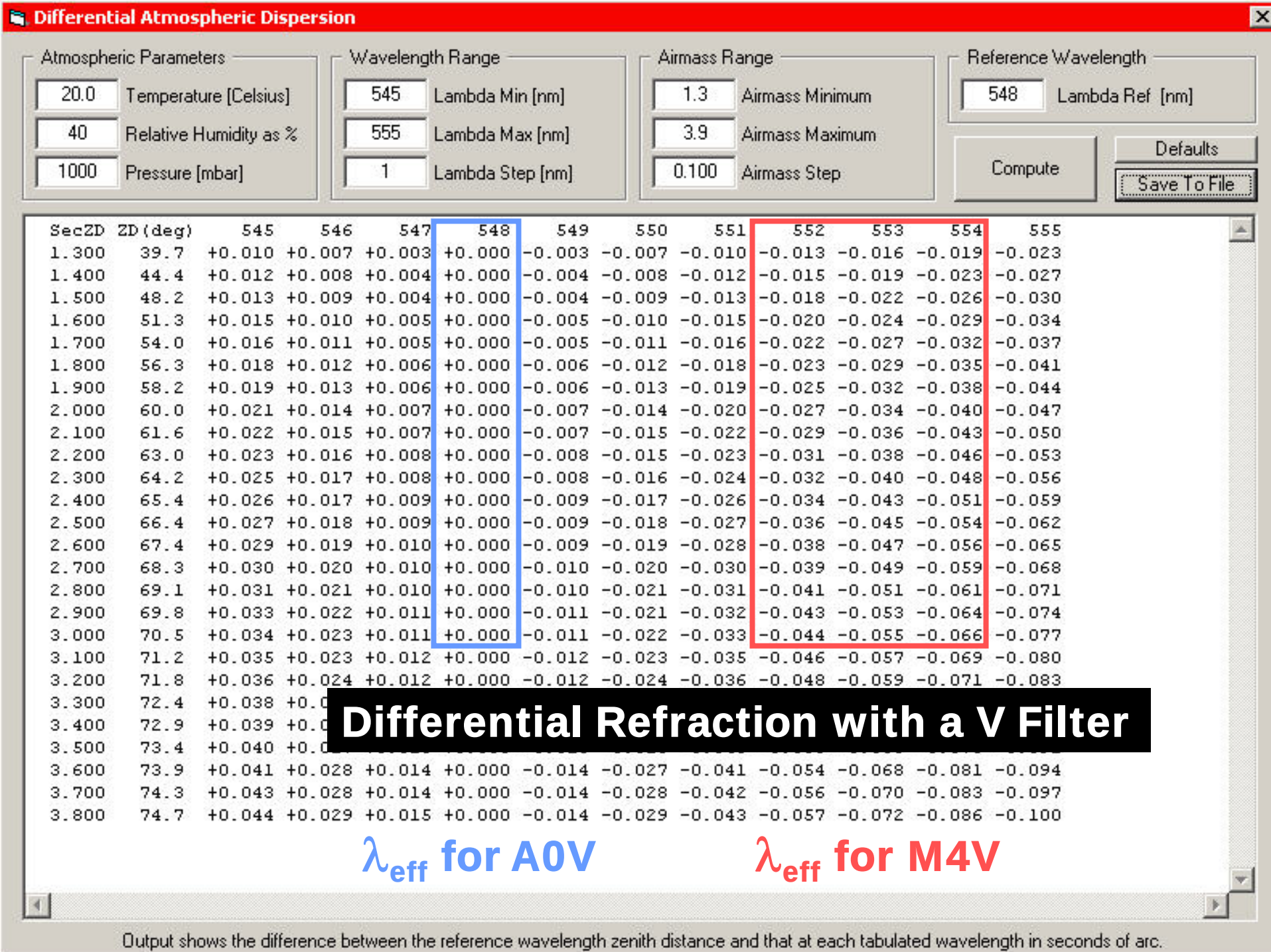
Output shows the difference between the reference wavelength zenith distance and that at each tabulated wavelength in seconds of arc.


Differential Atmospheric Dispersion
Differential Zenith Distance [arcsec]
10
8
6
4
2
0
-2
-4
1
1.5
2
2.5
3
3.5
4
4.5
5
Airmass [sec(zd)]
350 nm
400 nm
450 nm
500 nm
550 nm
600 nm
650 nm
700 nm
750 nm
800 nm
850 nm

Differential Refraction

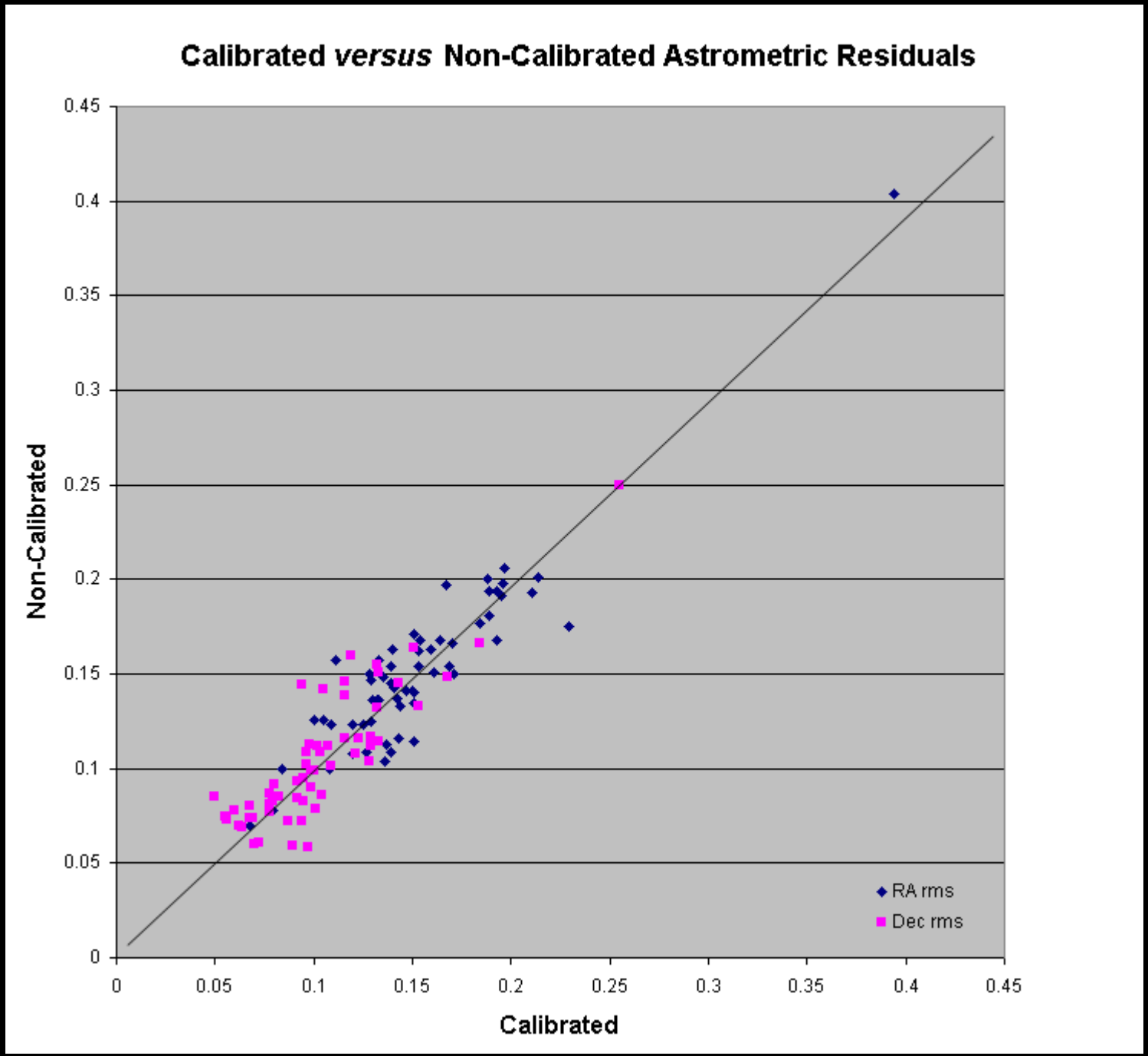
Barnard's Star Astrometry in V *versus* R

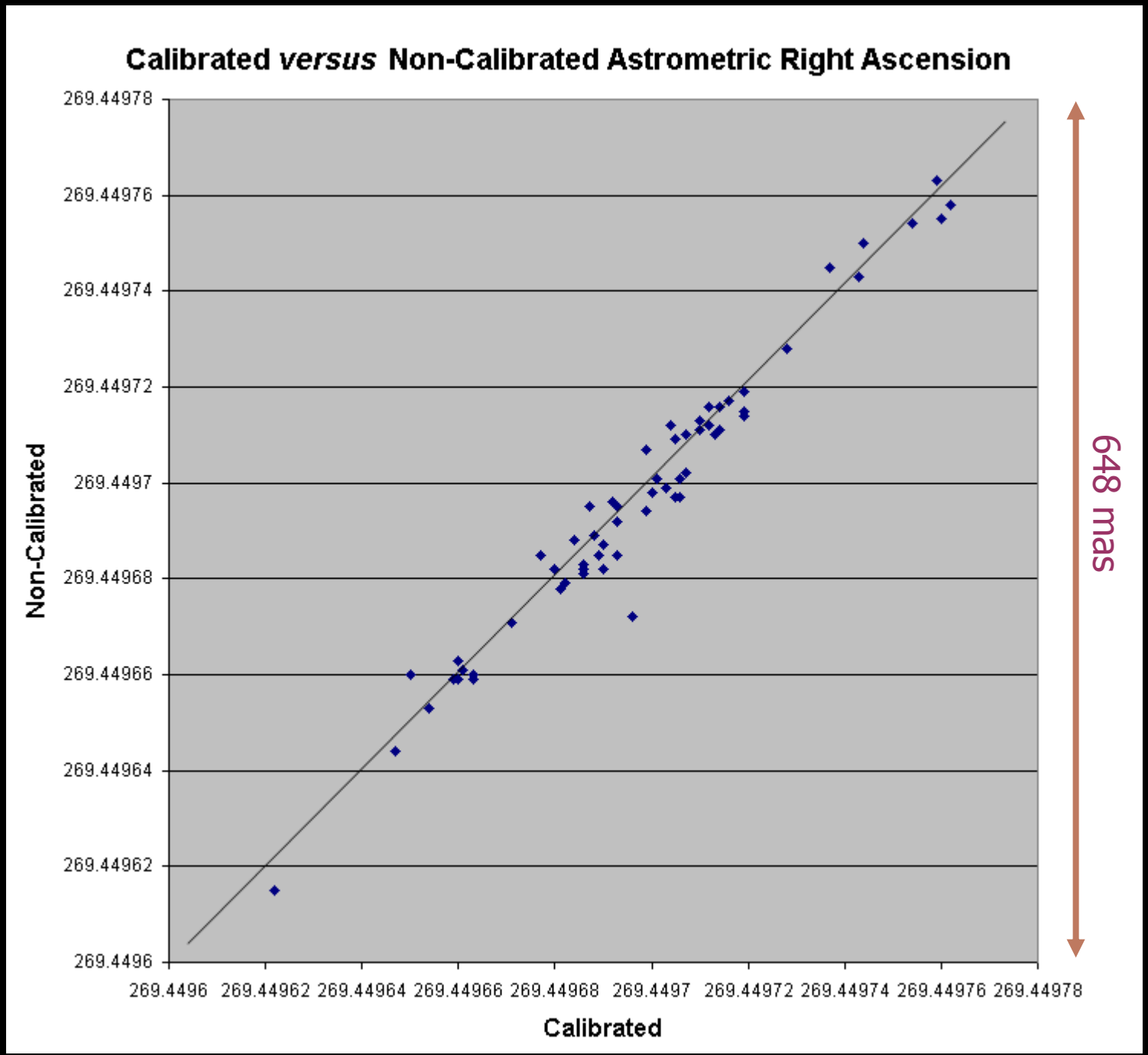


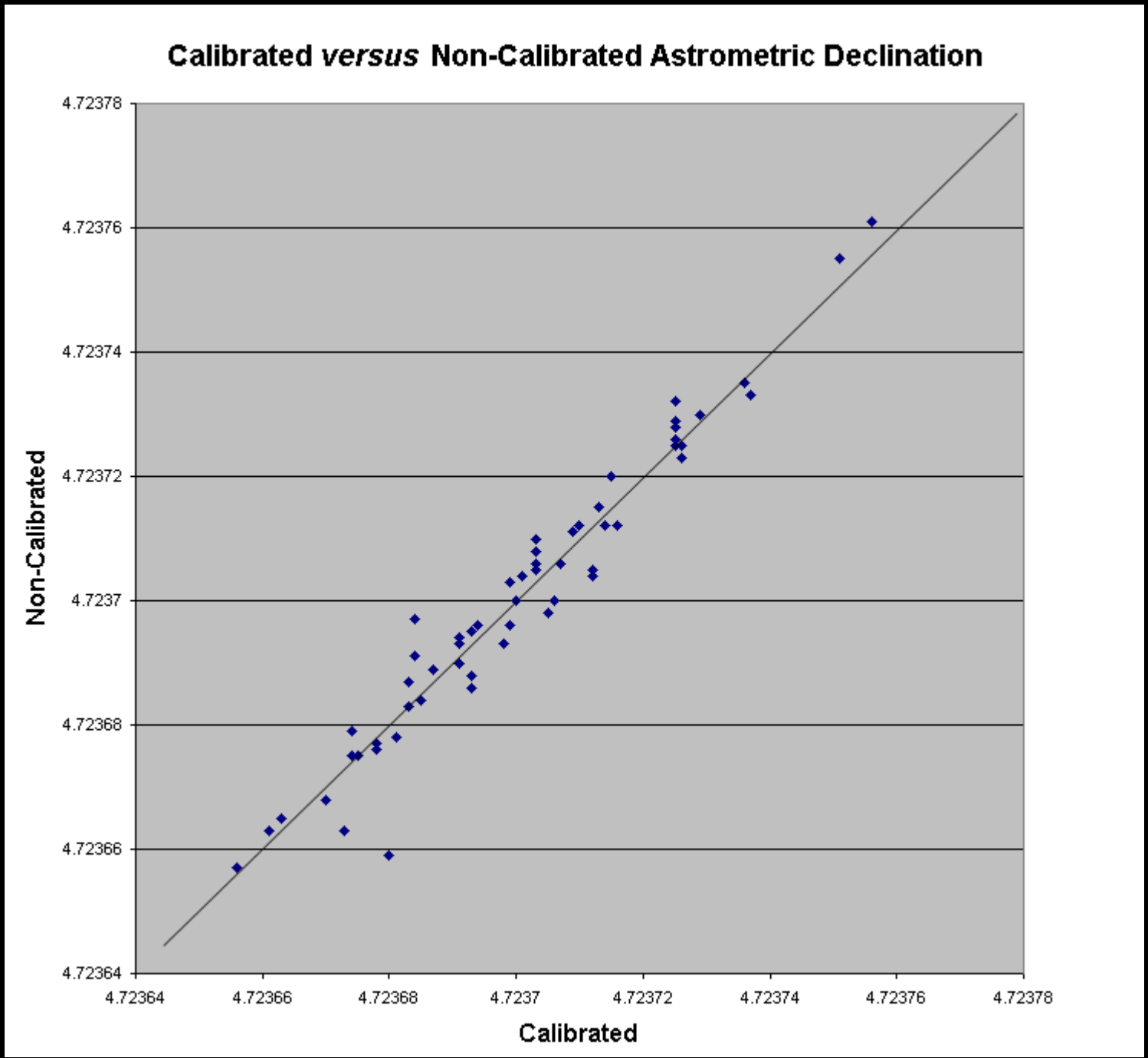

Differential Atmospheric Dispersion
Atmospheric Parameters
20.0
Temperature [Celsius]
40
Relative Humidity as %
1000
Pressure [mbar]
Wavelength Range
545
Lambda Min [nm]
555
Lambda Max [nm]
1
Lambda Step [nm]
Airmass Range
1.3
Airmass Minimum
3.9
Airmass Maximum
0.100
Airmass Step
Reference Wavelength
548
Lambda Ref [nm]
Compute
Defaults
Save To File
SecZD ZD(deg) 545 546 547 548 549 550 551 552 553 554 555
1.300 39.7 +0.010 +0.007 +0.003 +0.000 -0.003 -0.007 -0.010 -0.013 -0.016 -0.019 -0.023
1.400 44.4 +0.012 +0.008 +0.004 +0.000 -0.004 -0.008 -0.012 -0.015 -0.019 -0.023 -0.027
1.500 48.2 +0.013 +0.009 +0.004 +0.000 -0.004 -0.009 -0.013 -0.018 -0.022 -0.026 -0.030
1.600 51.3 +0.015 +0.010 +0.005 +0.000 -0.005 -0.010 -0.015 -0.020 -0.024 -0.029 -0.034
1.700 54.0 +0.016 +0.011 +0.005 +0.000 -0.005 -0.011 -0.016 -0.022 -0.027 -0.032 -0.037
1.800 56.3 +0.018 +0.012 +0.006 +0.000 -0.006 -0.012 -0.018 -0.023 -0.029 -0.035 -0.041
1.900 58.2 +0.019 +0.013 +0.006 +0.000 -0.006 -0.013 -0.019 -0.025 -0.032 -0.038 -0.044
2.000 60.0 +0.021 +0.014 +0.007 +0.000 -0.007 -0.014 -0.020 -0.027 -0.034 -0.040 -0.047
2.100 61.6 +0.022 +0.015 +0.007 +0.000 -0.007 -0.015 -0.022 -0.029 -0.036 -0.043 -0.050
2.200 63.0 +0.023 +0.016 +0.008 +0.000 -0.008 -0.015 -0.023 -0.031 -0.038 -0.046 -0.053
2.300 64.2 +0.025 +0.017 +0.008 +0.000 -0.008 -0.016 -0.024 -0.032 -0.040 -0.048 -0.056
2.400 65.4 +0.026 +0.017 +0.009 +0.000 -0.009 -0.017 -0.026 -0.034 -0.043 -0.051 -0.059
2.500 66.4 +0.027 +0.018 +0.009 +0.000 -0.009 -0.018 -0.027 -0.036 -0.045 -0.054 -0.062
2.600 67.4 +0.029 +0.019 +0.010 +0.000 -0.009 -0.019 -0.028 -0.038 -0.047 -0.056 -0.065
2.700 68.3 +0.030 +0.020 +0.010 +0.000 -0.010 -0.020 -0.030 -0.039 -0.049 -0.059 -0.068
2.800 69.1 +0.031 +0.021 +0.010 +0.000 -0.010 -0.021 -0.031 -0.041 -0.051 -0.061 -0.071
2.900 69.8 +0.033 +0.022 +0.011 +0.000 -0.011 -0.021 -0.032 -0.043 -0.053 -0.064 -0.074
3.000 70.5 +0.034 +0.023 +0.011 +0.000 -0.011 -0.022 -0.033 -0.044 -0.055 -0.066 -0.077
3.100 71.2 +0.035 +0.023 +0.012 +0.000 -0.012 -0.023 -0.035 -0.046 -0.057 -0.069 -0.080
3.200 71.8 +0.036 +0.024 +0.012 +0.000 -0.012 -0.024 -0.036 -0.048 -0.059 -0.071 -0.083
3.300 72.4 +0.038 +0.0
3.400 72.9 +0.039 +0.0
3.500 73.4 +0.040 +0.0
3.600 73.9 +0.041 +0.028 +0.014 +0.000 -0.014 -0.027 -0.041 -0.054 -0.068 -0.081 -0.094
3.700 74.3 +0.043 +0.028 +0.014 +0.000 -0.014 -0.028 -0.042 -0.056 -0.070 -0.083 -0.097
3.800 74.7 +0.044 +0.029 +0.015 +0.000 -0.014 -0.029 -0.043 -0.057 -0.072 -0.086 -0.100
Differential Refraction with a V Filter
λeff for A0V
λeff for M4V
Output shows the difference between the reference wavelength zenith distance and that at each tabulated wavelength in seconds of arc.

Does Image Calibration Matter?

- Do hot pixels affect astrometry?
- Always calibrate if possible, but…
  - Tested using 60 image sequence
  - Ran with and without dark subtraction
  - Compared coordinates
  - Compared astrometric residuals


Calibrated versus Non-Calibrated Astrometric Residuals
Non-Calibrated
Calibrated
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
RA rms
Dec rms

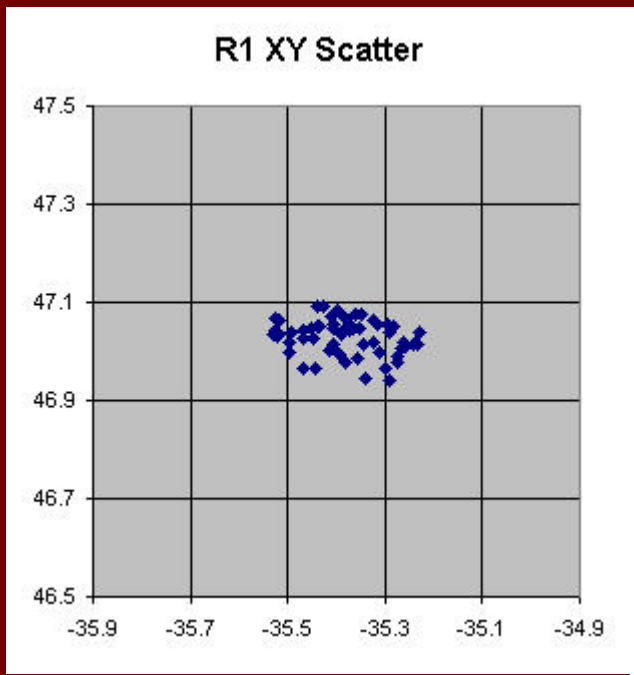

Calibrated *versus* Non-Calibrated Astrometric Right Ascension
269.44978
269.44976
269.44974
269.44972
269.4497
269.44968
269.44966
269.44964
269.44962
269.4496
Non-Calibrated
269.4496
269.44962
269.44964
269.44966
269.44968
269.4497
269.44972
269.44974
269.44976
269.44978
Calibrated
648 mas

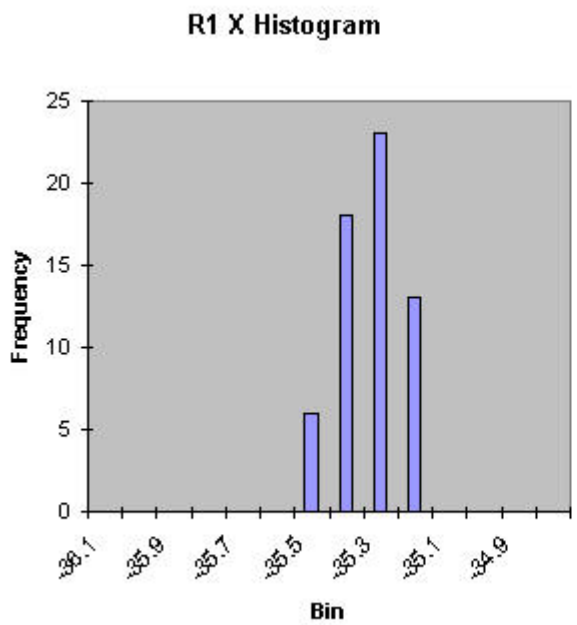

Calibrated versus Non-Calibrated Astrometric Declination
Non-Calibrated
4.72378
4.72376
4.72374
4.72372
4.7237
4.72368
4.72366
4.72364
4.72364
4.72366
4.72368
4.7237
4.72372
4.72374
4.72376
4.72378
Calibrated

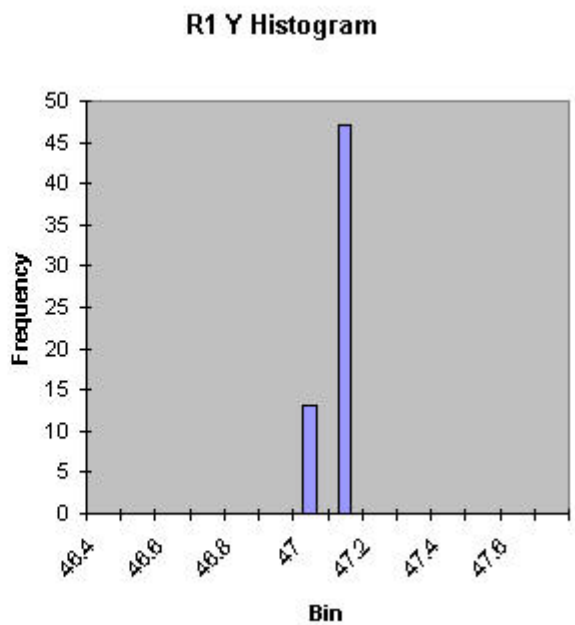
Star Image Centroid Error

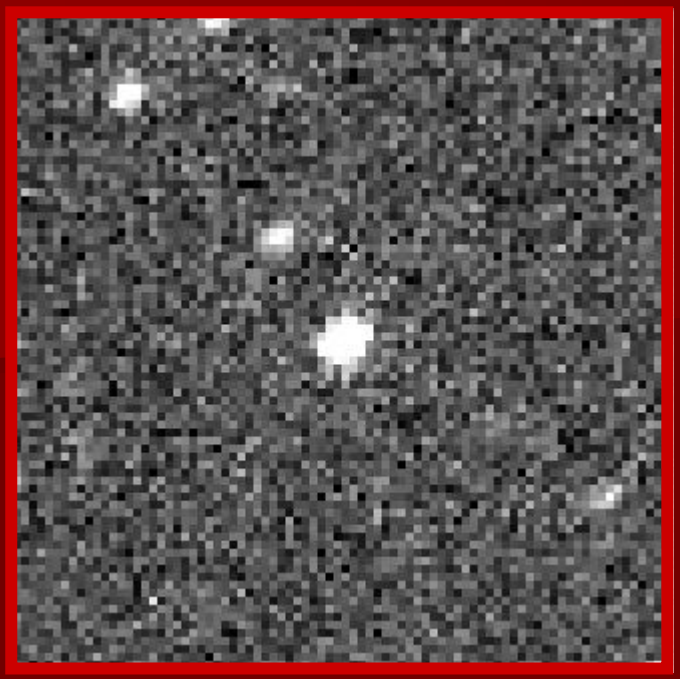
- What is the error distribution of centroids?
  - Collection of pixels > threshold ADU value
  - Photon statistics $\rightarrow$ normal distribution?
- How does the error depend on brightness?
  - Expectation: Dim star $\rightarrow$ larger σ ?
  - What is the faintest useful reference star?
- Centroids measured differentially w.r.t. Barnard's Star

- R1 (*i.e.*, ref star 1)
- m_V=11.45

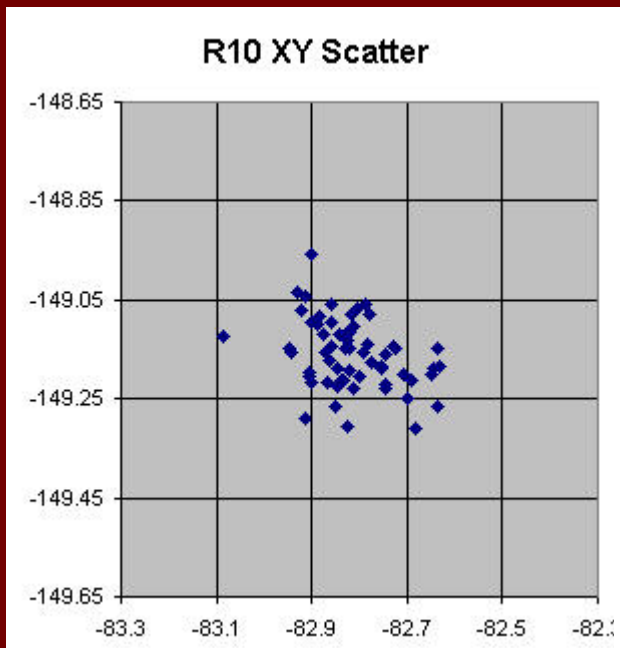


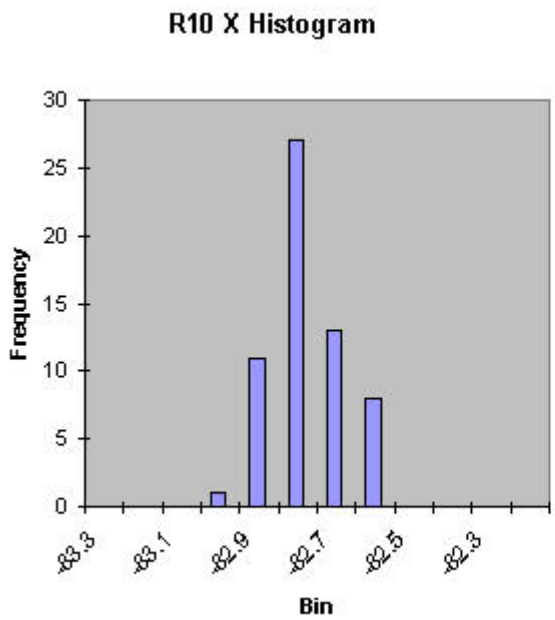


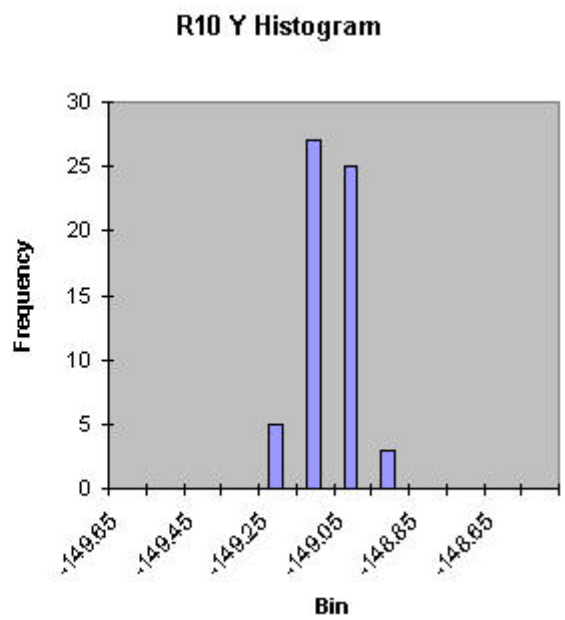


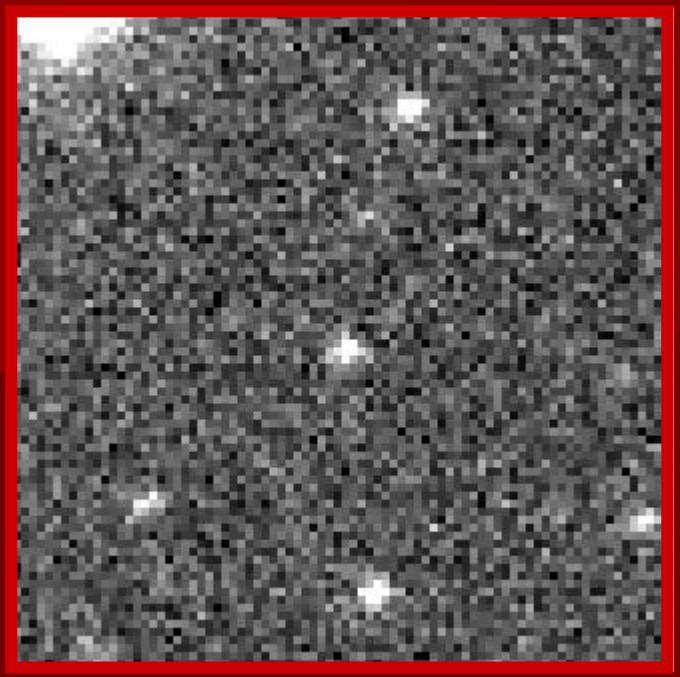


- R10
- m_V=13.55

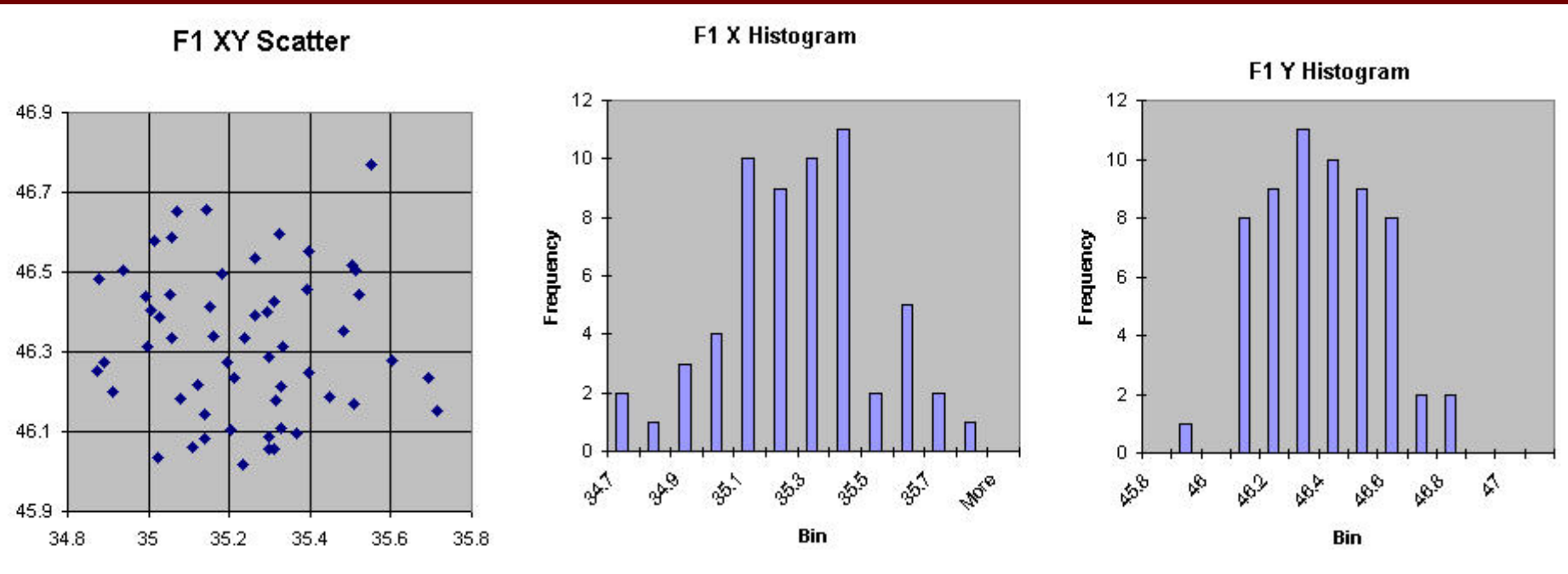

R10 XY Scatter
-148.65
-148.85
-149.05
-149.25
-149.45
-149.65
-83.3
-83.1
-82.9
-82.7
-82.5

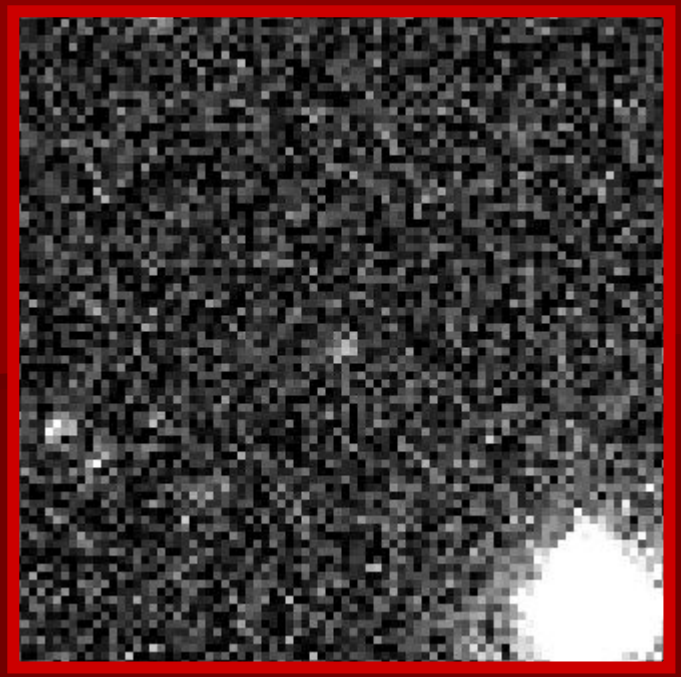

R10 X Histogram
Frequency
30
25
20
15
10
5
0
-83.3
-83.1
-82.9
-82.7
-82.5
-82.3
Bin


R10 Y Histogram
Frequency
30
25
20
15
10
5
0
-149.65
-149.45
-149.25
-149.05
-148.85
-148.65
Bin

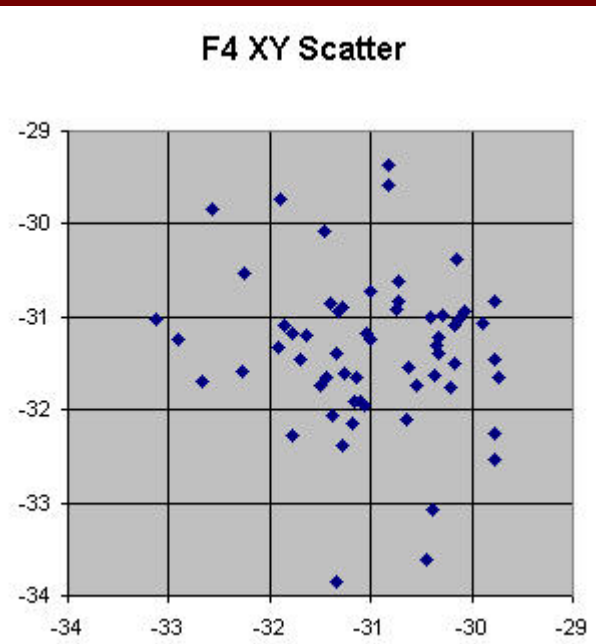


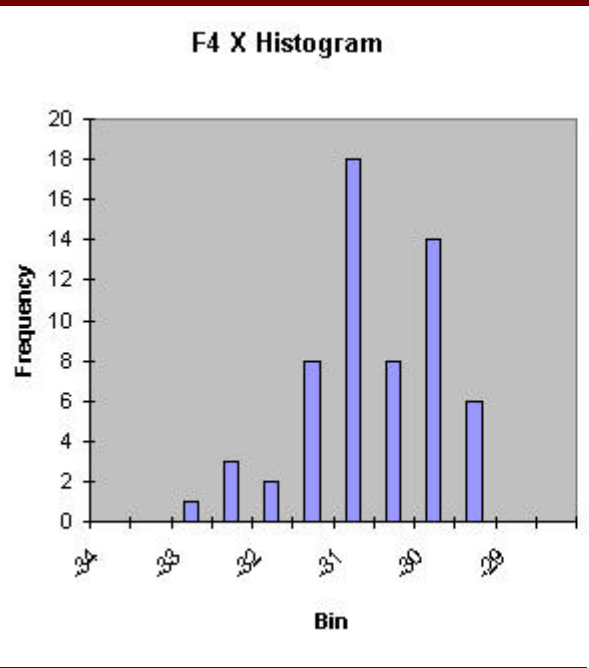
- F1 (*i.e.*, faint star 1)
- m_V=15.4

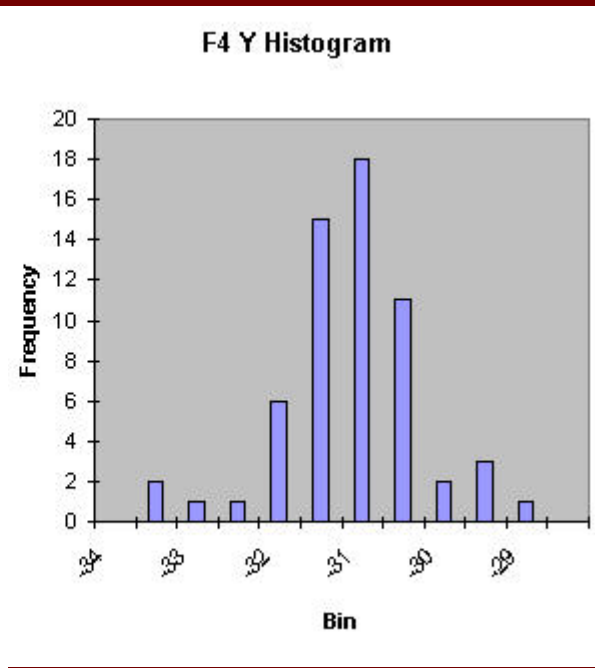


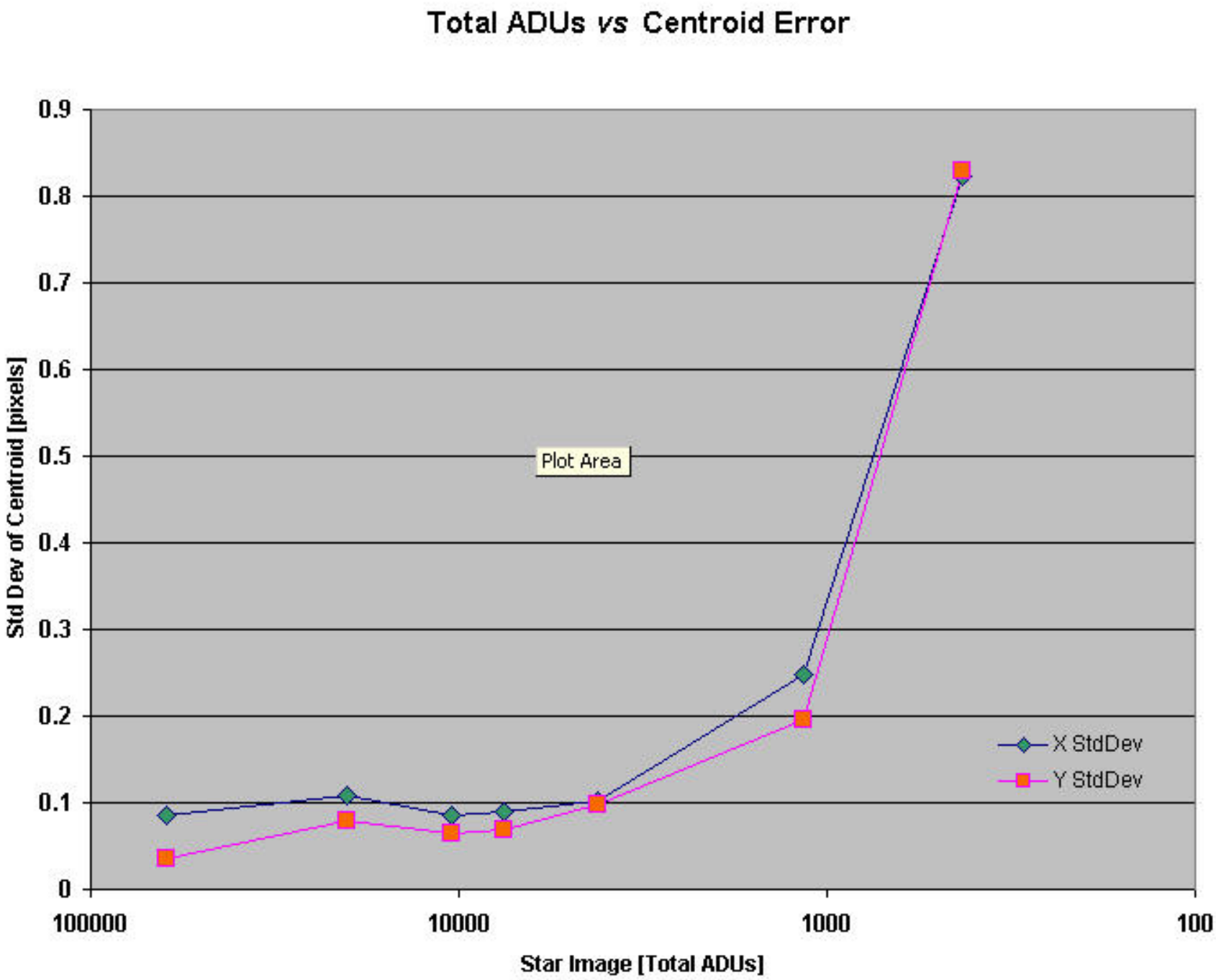


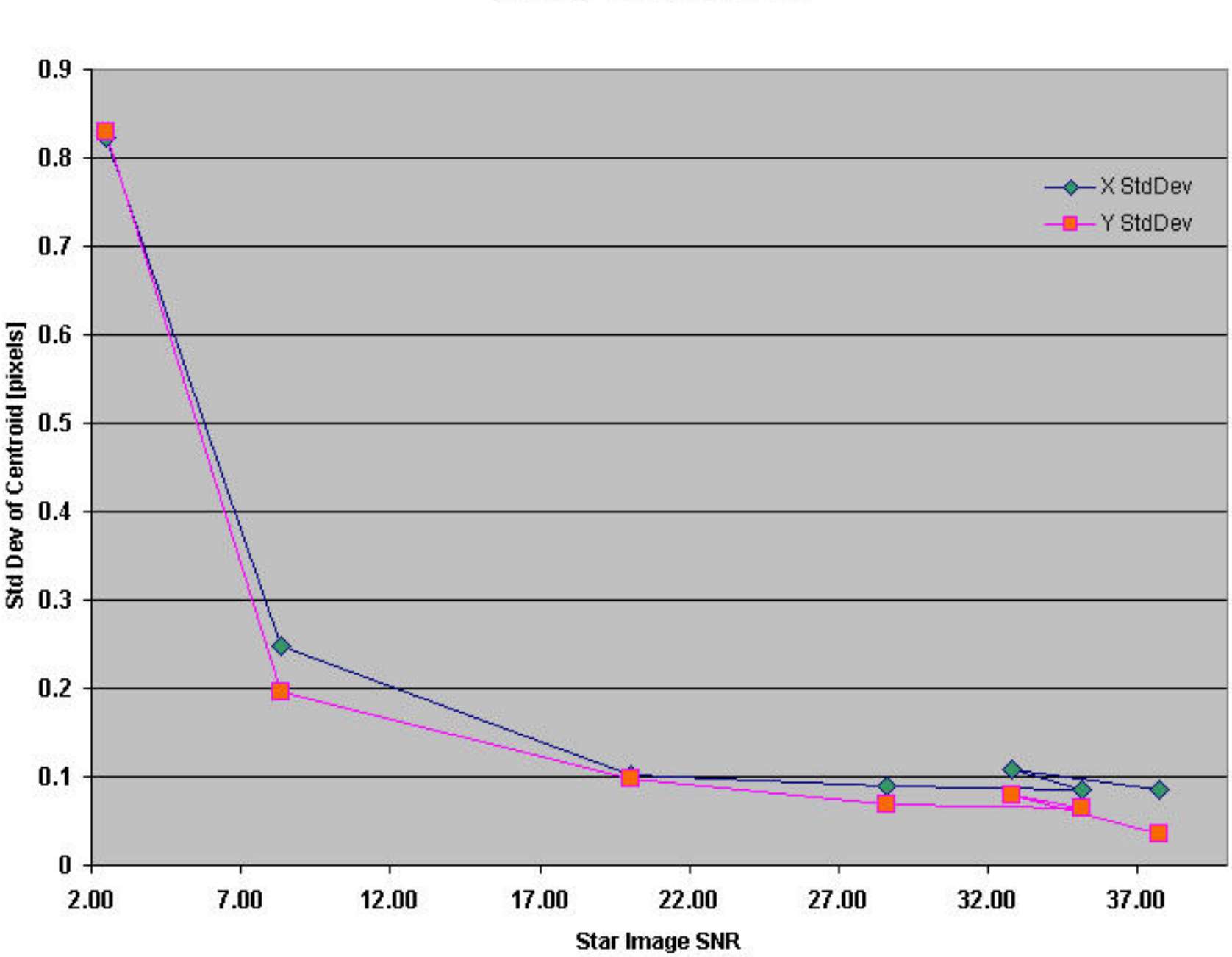
- F4
- m_V=16.6

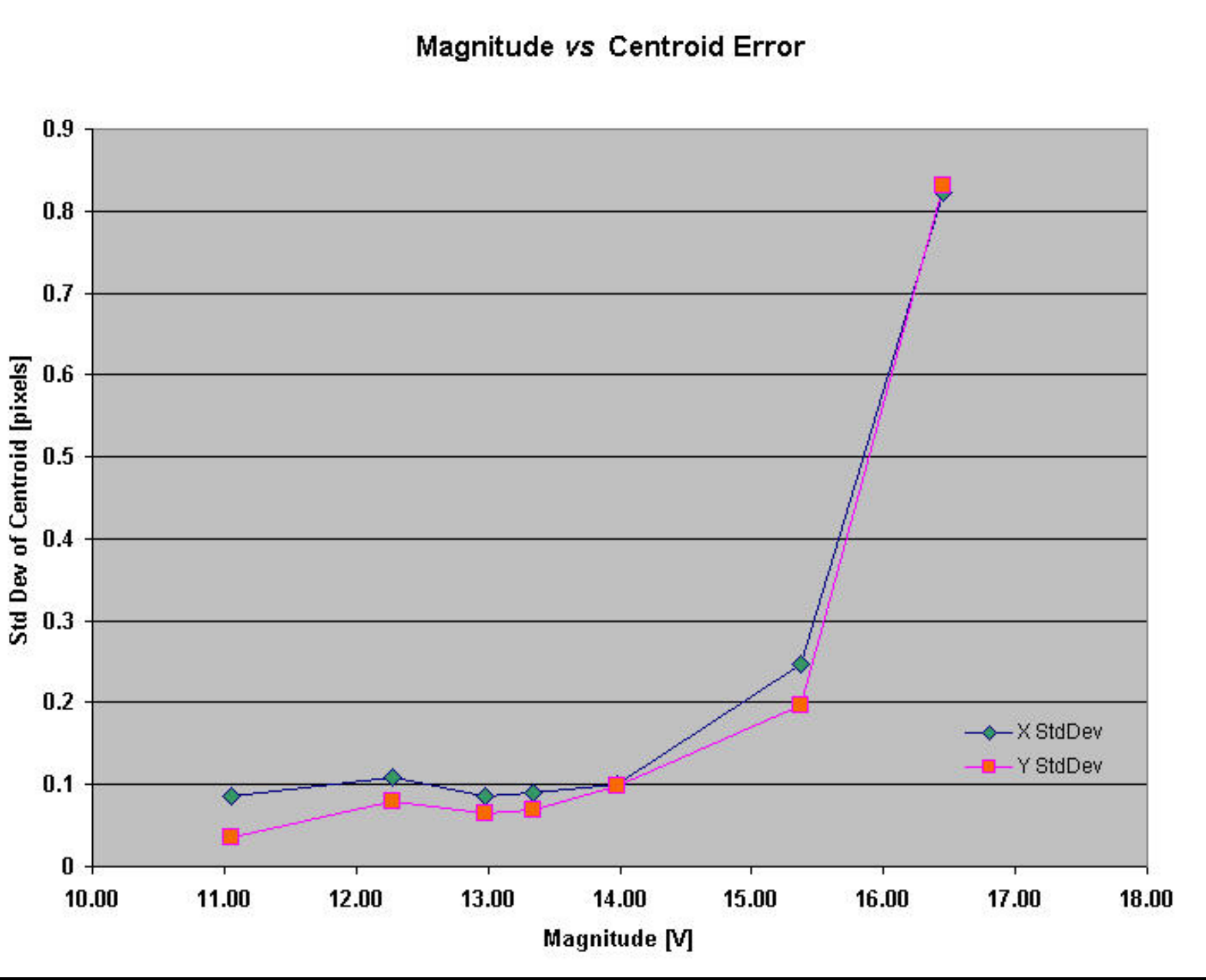

F4 XY Scatter
-29
-30
-31
-32
-33
-34
-34
-33
-32
-31
-30
-29


F4 X Histogram
Frequency
20
18
16
14
12
10
8
6
4
2
0
34
33
32
31
30
29
Bin


F4 Y Histogram
Frequency
20
18
16
14
12
10
8
6
4
2
0
34
33
32
31
30
29
Bin

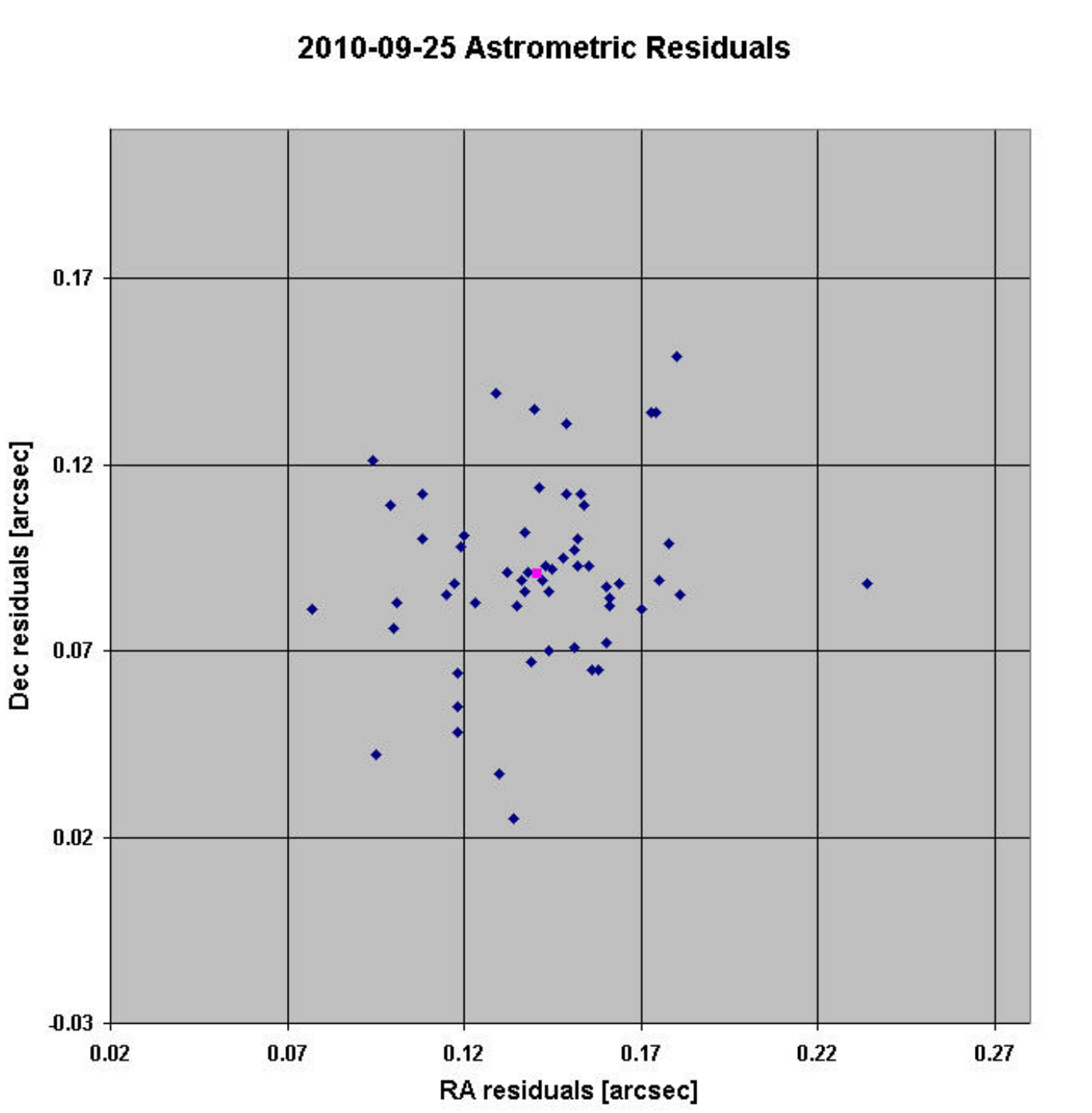

Total ADUs vs Centroid Error
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Std Dev of Centroid [pixels]
Plot Area
X StdDev
Y StdDev
100000
10000
1000
100
Star Image [Total ADUs]


SNR vs Centroid Error
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Std Dev of Centroid [pixels]
X StdDev
Y StdDev
2.00
7.00
12.00
17.00
22.00
27.00
32.00
37.00
Star Image SNR

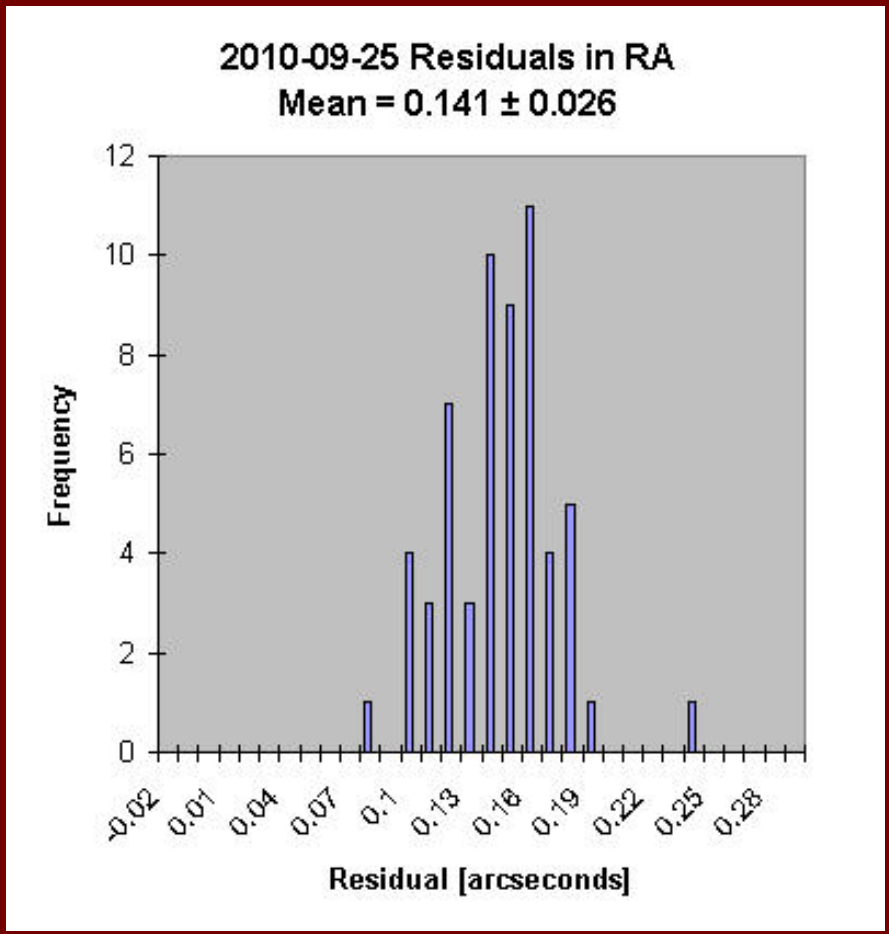

Magnitude vs Centroid Error
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Std Dev of Centroid [pixels]
10.00
11.00
12.00
13.00
14.00
15.00
16.00
17.00
18.00
Magnitude [V]
X StdDev
Y StdDev

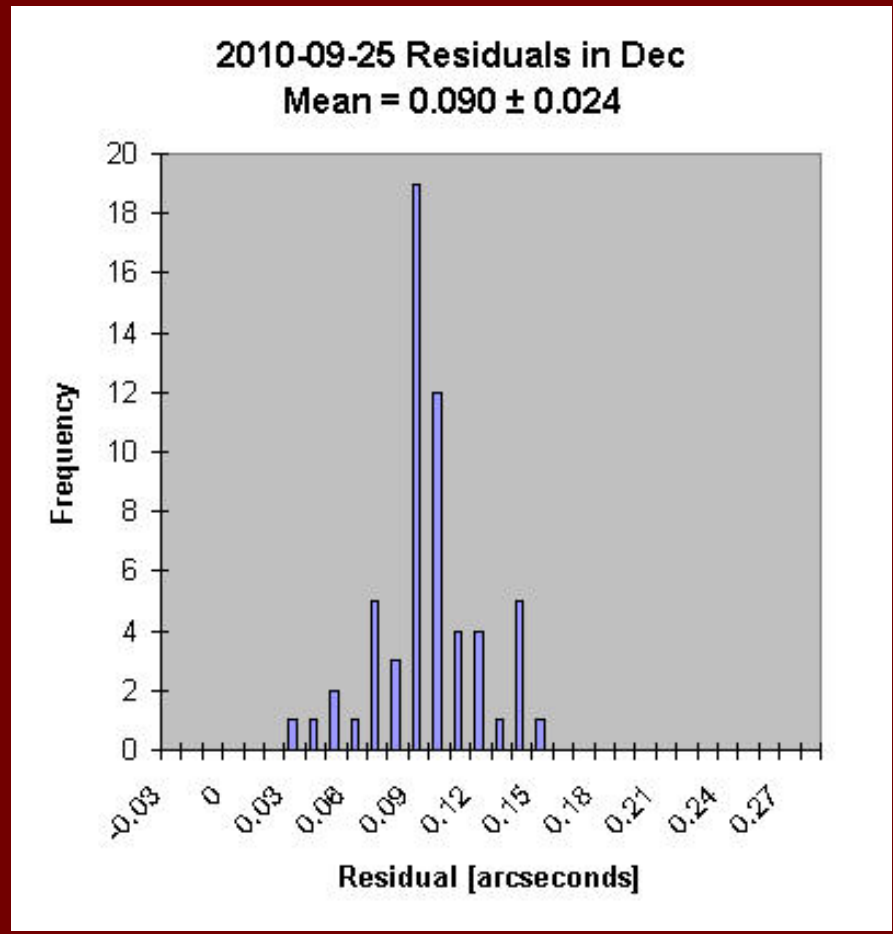
Astrometric Solution

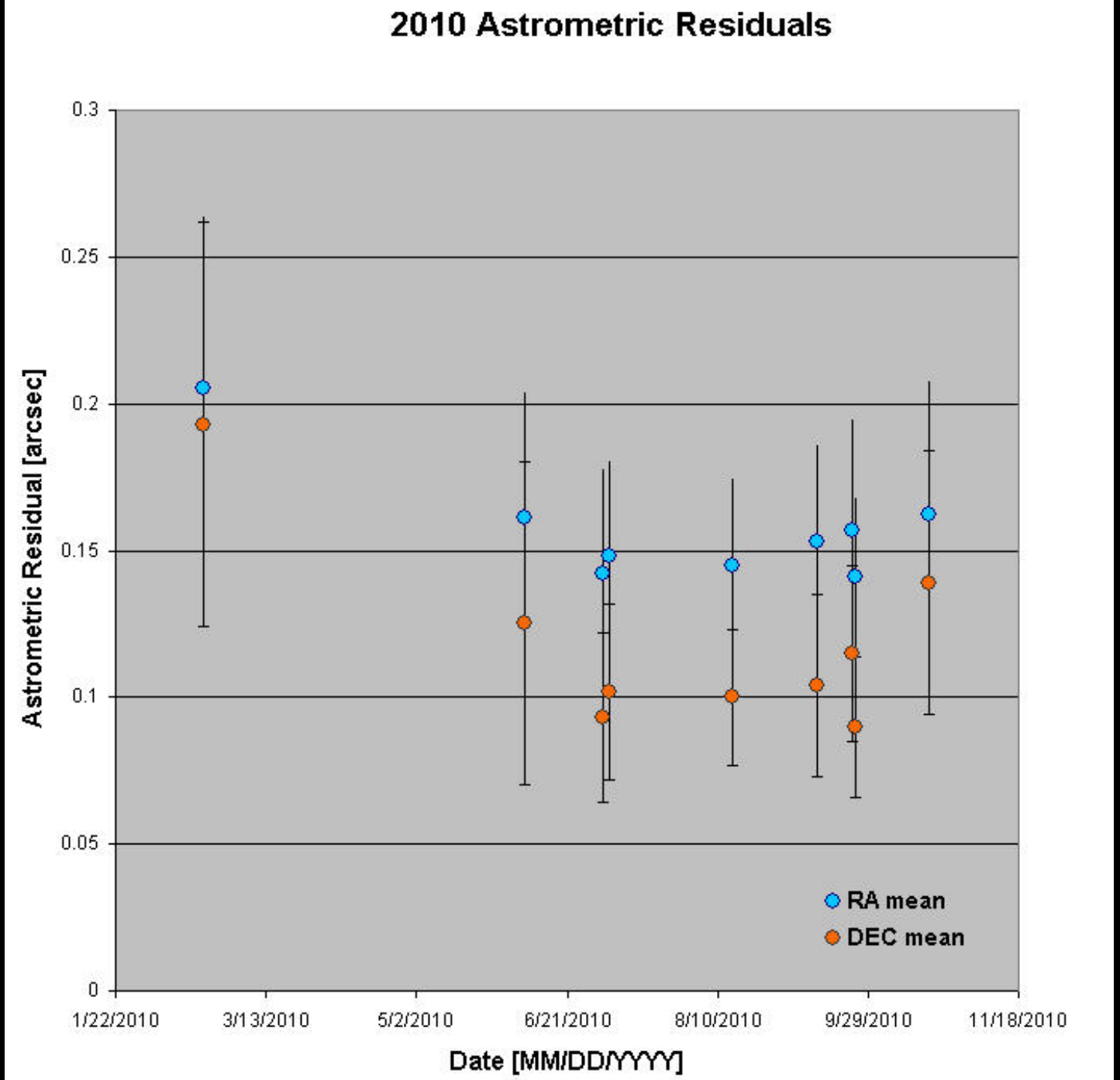
- Key idea:
  - Always use same set of reference stars
  - Astrometric solution always “the same”
  - Centroid errors will alter residuals
  - Residuals will cluster around mean value
  - Distribution will be normal distribution

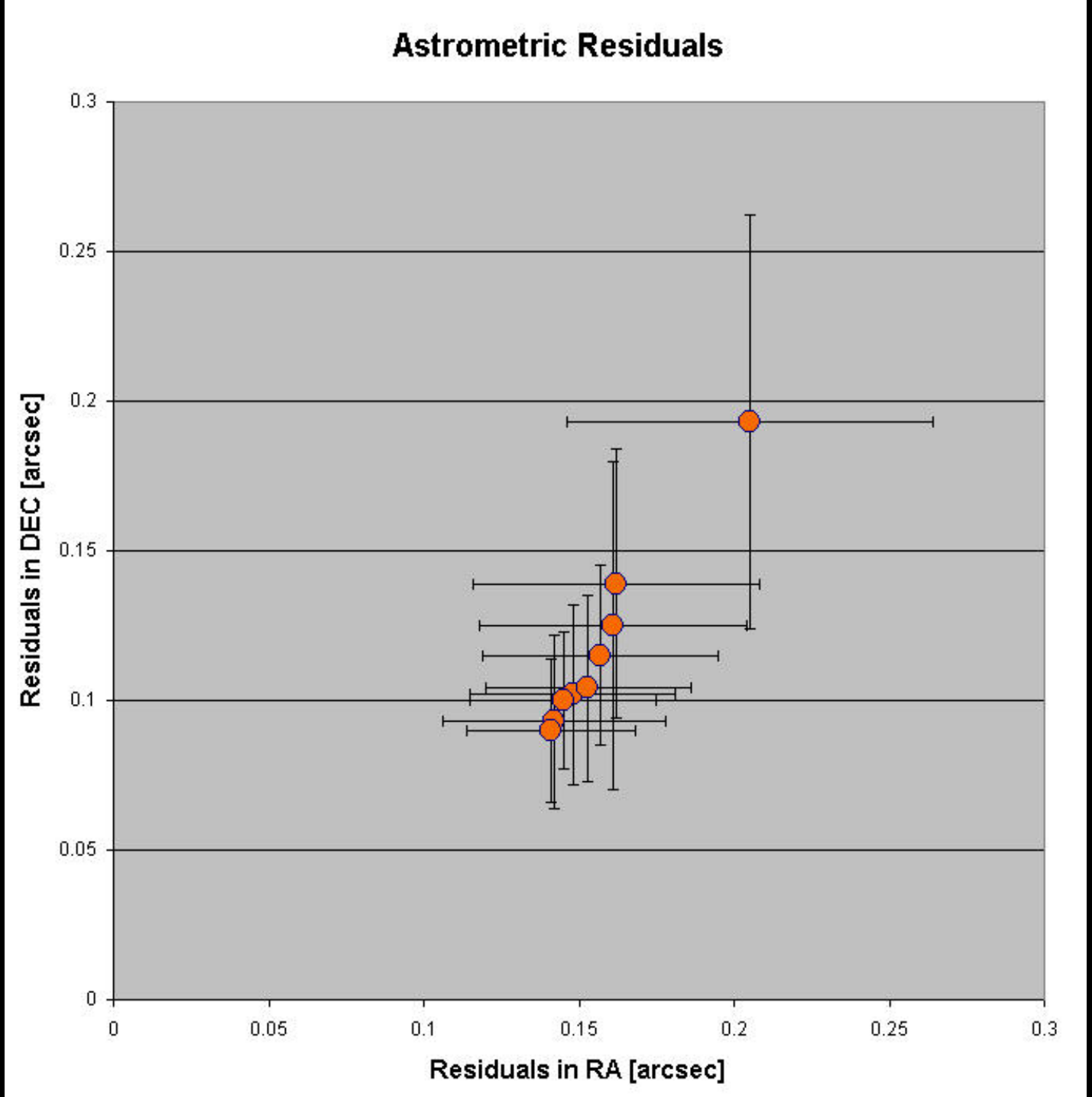

2010-09-25 Astrometric Residuals
0.17
0.12
0.07
0.02
-0.03
Dec residuals [arcsec]
0.02
0.07
0.12
0.17
0.22
0.27
RA residuals [arcsec]

Distribution of Residuals






2010 Astrometric Residuals
Astrometric Residual [arcsec]
0.3
0.25
0.2
0.15
0.1
0.05
0
1/22/2010
3/13/2010
5/2/2010
6/21/2010
8/10/2010
9/29/2010
11/18/2010
Date [MM/DD/YYYY]
RA mean
DEC mean

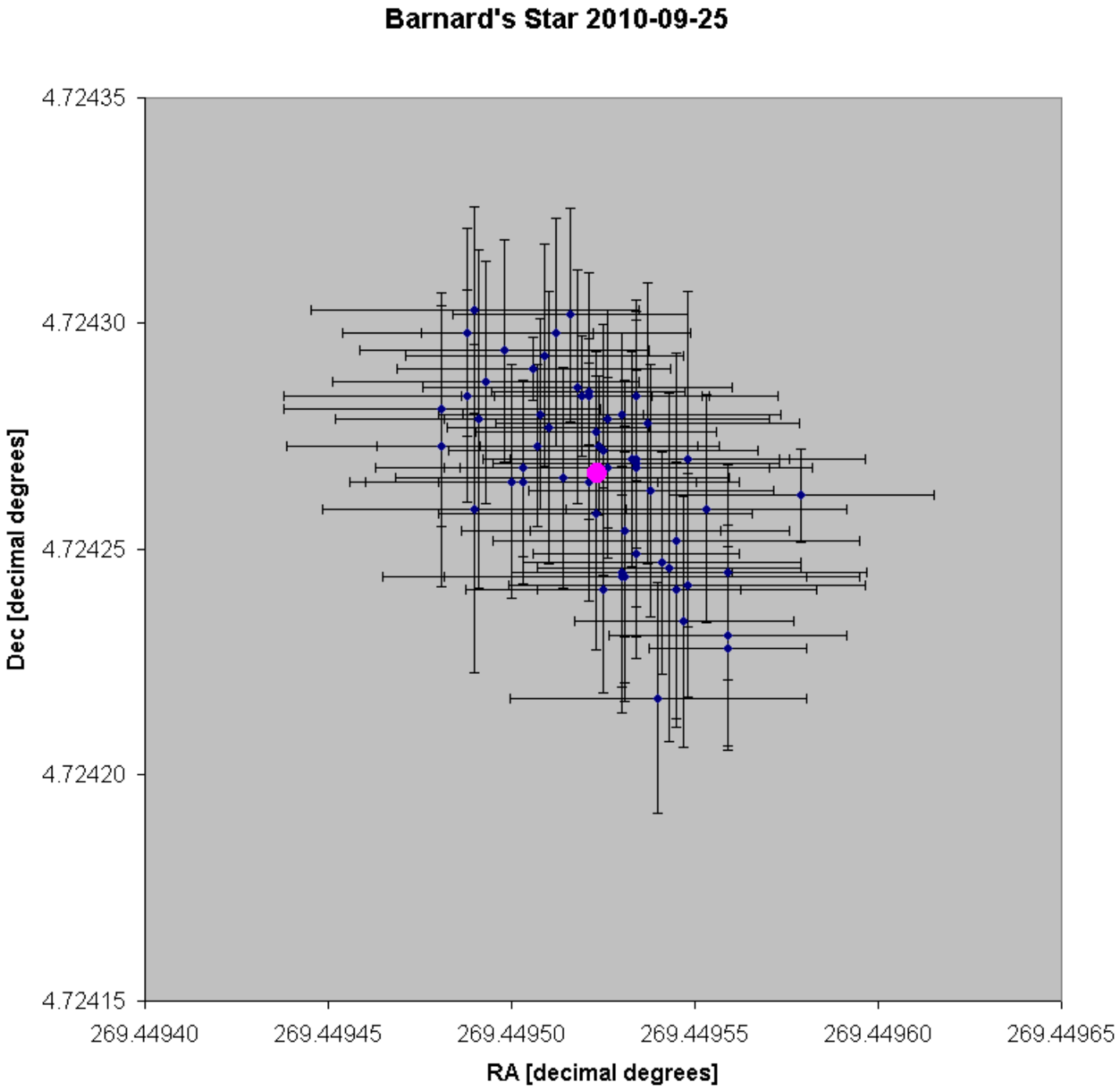

Astrometric Residuals
Residuals in DEC [arcsec]
Residuals in RA [arcsec]
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.05
0.1
0.15
0.2
0.25
0.3

Astrometric Position

- Key idea:
  - Astrometric solution always "the same"
  - Positions tend toward a mean value
  - Distribution will be normal distribution
- Tests:
  - Measure sets of 60 images
  - Compute standard deviation of position
- Standard error = $\sigma/\sqrt{N}$

Astrometric Data Report

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	AIP4Win v2.3.33 Magnitude Measurement Tool												
2	Astrometric Coordinates in Text Format												
3	Reference stars = 11												
4	Now = 10/8/2010 9:48:31 PM												
5	Seq#	Julian Day	Focal[mm]	PA[d.d]	X[pix]	Y[pix]	RA[d.d]	Dec[d.d]	RArms	DCrms	HH MM SS.ss	+DD MM SS.s	FileName
6	0	2455465.4719	911.0738	174.8909	1109.1150	724.255	269.449540	4.724217	0.145	0.092	17 57 47.89	+04 43 27.2	Barnstar-001-V60s.fit
7	1	2455465.4729	911.1903	174.9034	1110.3390	725.270	269.449559	4.724245	0.137	0.086	17 57 47.89	+04 43 27.3	Barnstar-002-V60s.fit
8	2	2455465.4738	911.0766	174.8860	1110.7570	725.883	269.449519	4.724284	0.118	0.048	17 57 47.88	+04 43 27.4	Barnstar-003-V60s.fit
9	3	2455465.4748	911.1803	174.8925	1111.6590	726.666	269.449526	4.724268	0.160	0.072	17 57 47.89	+04 43 27.4	Barnstar-004-V60s.fit
10	4	2455465.4758	911.2407	174.8792	1112.4750	726.338	269.449534	4.724249	0.101	0.083	17 57 47.89	+04 43 27.3	Barnstar-005-V60s.fit
11	5	2455465.4767	910.8706	174.8941	1113.0990	726.950	269.449537	4.724278	0.149	0.112	17 57 47.89	+04 43 27.4	Barnstar-006-V60s.fit
12	6	2455465.4777	910.8468	174.8869	1113.2640	726.589	269.449533	4.724270	0.144	0.086	17 57 47.89	+04 43 27.4	Barnstar-007-V60s.fit
13	7	2455465.4787	910.9625	174.8899	1114.2280	726.405	269.449547	4.724234	0.108	0.100	17 57 47.89	+04 43 27.2	Barnstar-008-V60s.fit
14	8	2455465.4796	920.5098	173.9535	1114.7880	726.566	269.449874	4.724502	4.913	3.134	17 57 47.97	+04 43 28.2	Barnstar-009-V60s.fit
15	9	2455465.4806	911.0141	174.8864	1115.2200	726.386	269.449559	4.724228	0.077	0.081	17 57 47.89	+04 43 27.2	Barnstar-010-V60s.fit
16	10	2455465.4816	911.1652	174.8776	1115.4930	726.321	269.449523	4.724258	0.154	0.109	17 57 47.89	+04 43 27.3	Barnstar-011-V60s.fit
17	11	2455465.4825	911.1594	174.8983	1115.6550	726.408	269.449525	4.724241	0.135	0.082	17 57 47.89	+04 43 27.3	Barnstar-012-V60s.fit
18	12	2455465.4835	910.9489	174.8854	1115.9300	726.182	269.449530	4.724245	0.108	0.112	17 57 47.89	+04 43 27.3	Barnstar-013-V60s.fit
19	13	2455465.4845	910.8863	174.8830	1116.3670	726.080	269.449521	4.724284	0.119	0.098	17 57 47.89	+04 43 27.4	Barnstar-014-V60s.fit
20	14	2455465.4855	910.8912	174.8717	1116.3460	725.982	269.449526	4.724279	0.160	0.087	17 57 47.89	+04 43 27.4	Barnstar-015-V60s.fit
21	15	2455465.4864	911.0373	174.8802	1116.6080	726.036	269.449510	4.724277	0.099	0.109	17 57 47.88	+04 43 27.4	Barnstar-016-V60s.fit
22	16	2455465.4874	911.0205	174.8933	1116.8730	725.895	269.449500	4.724265	0.143	0.093	17 57 47.88	+04 43 27.4	Barnstar-017-V60s.fit
23	17	2455465.4884	911.1496	174.8793	1117.4670	725.950	269.449506	4.724290	0.134	0.025	17 57 47.88	+04 43 27.4	Barnstar-018-V60s.fit
24	18	2455465.4893	910.9007	174.8687	1117.2820	726.238	269.449525	4.724272	0.152	0.100	17 57 47.89	+04 43 27.4	Barnstar-019-V60s.fit
25	19	2455465.4903	911.0019	174.8688	1117.9900	726.291	269.449579	4.724262	0.130	0.037	17 57 47.90	+04 43 27.3	Barnstar-020-V60s.fit
26	20	2455465.4913	911.3158	174.8904	1117.8650	726.206	269.449553	4.724259	0.138	0.091	17 57 47.89	+04 43 27.3	Barnstar-021-V60s.fit
27	21	2455465.4922	910.8112	174.8790	1118.2040	726.087	269.449518	4.724286	0.152	0.093	17 57 47.88	+04 43 27.4	Barnstar-022-V60s.fit
28	22	2455465.4932	910.9561	174.8623	1118.4220	725.857	269.449516	4.724302	0.115	0.085	17 57 47.88	+04 43 27.5	Barnstar-023-V60s.fit
29	23	2455465.4942	911.1515	174.8843	1118.6280	725.246	269.449493	4.724287	0.151	0.097	17 57 47.88	+04 43 27.4	Barnstar-024-V60s.fit
30	24	2455465.4951	910.7821	174.8772	1119.1060	725.354	269.449541	4.724247	0.136	0.089	17 57 47.89	+04 43 27.3	Barnstar-025-V60s.fit
31	25	2455465.4961	910.8367	174.8403	1119.3980	725.095	269.449498	4.724294	0.142	0.089	17 57 47.88	+04 43 27.5	Barnstar-026-V60s.fit
32	26	2455465.4971	910.9794	174.8579	1119.2390	725.054	269.449512	4.724298	0.132	0.091	17 57 47.88	+04 43 27.5	Barnstar-027-V60s.fit
33	27	2455465.4980	911.0343	174.8766	1119.6120	724.681	269.449508	4.724280	0.100	0.076	17 57 47.88	+04 43 27.4	Barnstar-028-V60s.fit
34	28	2455465.4990	911.1055	174.9062	1120.0520	724.224	269.449503	4.724268	0.144	0.070	17 57 47.88	+04 43 27.4	Barnstar-029-V60s.fit
35	29	2455465.5000	911.0400	174.8752	1120.6640	724.107	269.449534	4.724269	0.141	0.114	17 57 47.89	+04 43 27.4	Barnstar-030-V60s.fit
36	30	2455465.5009	911.1117	174.8649	1120.3910	724.219	269.449534	4.724270	0.151	0.071	17 57 47.89	+04 43 27.4	Barnstar-031-V60s.fit
37	31	2455465.5019	910.9768	174.8785	1121.4030	724.389	269.449531	4.724254	0.094	0.121	17 57 47.89	+04 43 27.3	Barnstar-032-V60s.fit
38	32	2455465.5029	910.8195	174.8627	1121.0790	724.098	269.449534	4.724268	0.173	0.134	17 57 47.89	+04 43 27.4	Barnstar-033-V60s.fit
39	33	2455465.5038	910.9042	174.8599	1121.6070	723.931	269.449509	4.724293	0.136	0.089	17 57 47.88	+04 43 27.5	Barnstar-034-V60s.fit
40	34	2455465.5048	911.0821	174.8755	1121.8740	723.437	269.449559	4.724231	0.117	0.088	17 57 47.89	+04 43 27.2	Barnstar-035-V60s.fit

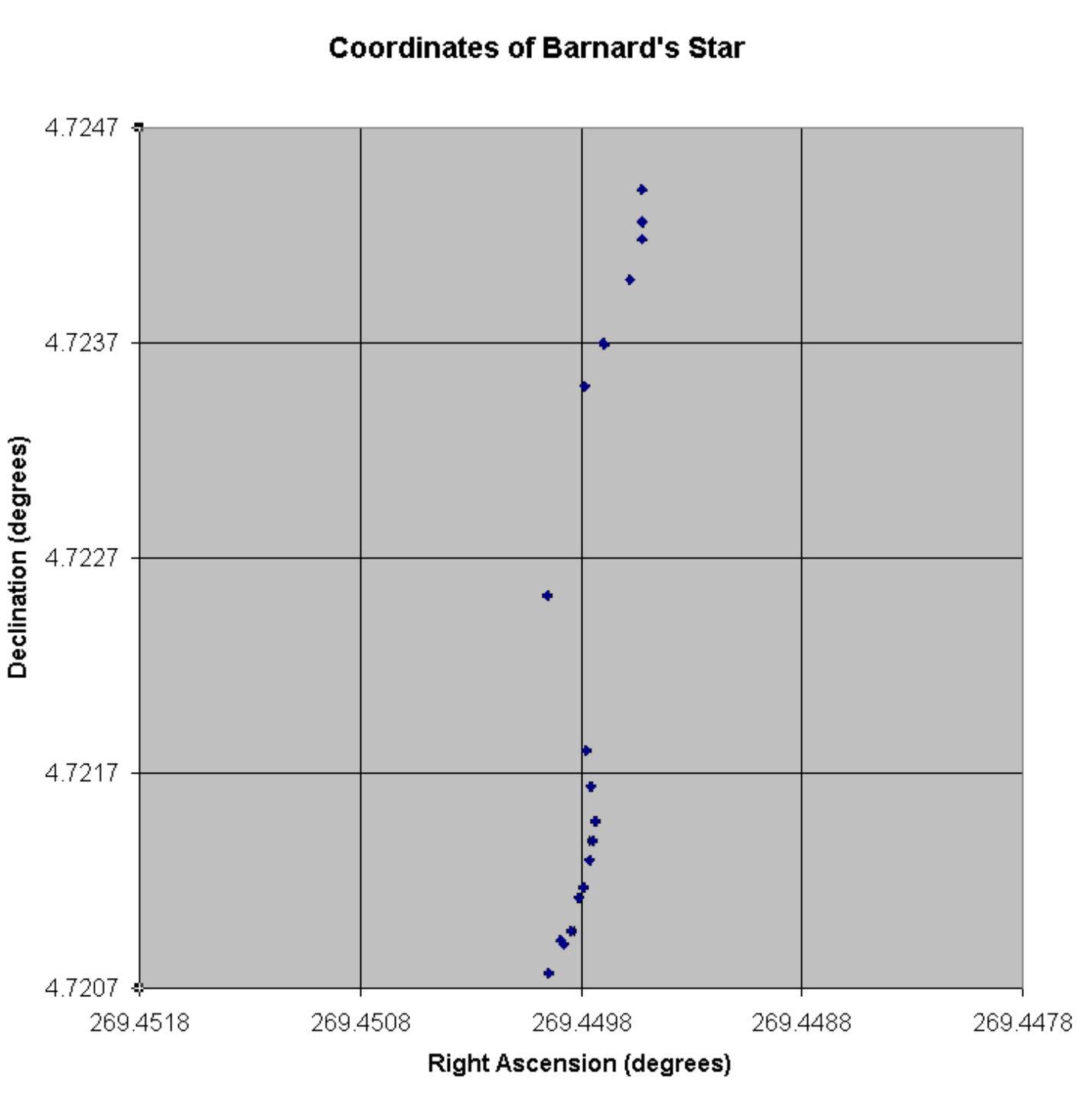

Barnard's Star 2010-09-25
4.72435
4.72430
4.72425
4.72420
4.72415
Dec [decimal degrees]
269.44940
269.44945
269.44950
269.44955
269.44960
269.44965
RA [decimal degrees]

Results for Barnard's Star

Barnard's Star

Mean Positions and Standard Errors for 20 Nights in 2009-2010

Date	JD	RA	Dec	RA_{ste}	Dec_{ste}	N
2009-06-27	2455010.81407	269.44994707	4.72076871	0.00000388	0.00000576	14.00
2009-07-16	2455028.80895	269.44987810	4.72090612	0.00000165	0.00000165	41.00
2009-07-17	2455030.80288	269.44989435	4.72092106	0.00000354	0.00000189	31.00
2009-07-24	2455037.76499	269.44984252	4.72096489	0.00000846	0.00000717	27.00
2009-08-16	2455060.70775	269.44980920	4.72112190	0.00000819	0.00000548	20.00
2009-08-23	2455067.71629	269.44978710	4.72116720	0.00000521	0.00000493	40.00
2009-09-10	2455085.65938	269.44975914	4.72129771	0.00000478	0.00000506	35.00
2009-09-25	2455100.68891	269.44975095	4.72138550	0.00000908	0.00000780	20.00
2009-10-06	2455111.61915	269.44973775	4.72147455	0.00000709	0.00000671	20.00
2009-11-01	2455137.60667	269.44975657	4.72163799	0.00000473	0.00000389	92.00
2009-11-24	2455160.57452	269.44977870	4.72180378	0.00000514	0.00000480	40.00
2010-02-20	2455249.06188	269.44995185	4.72252358	0.00000398	0.00000548	40.00
2010-06-07	2455355.80518	269.44978542	4.72349878	0.00000340	0.00000284	36.00
2010-07-03	2455381.78179	269.44969725	4.72369041	0.00000509	0.00000338	32.00
2010-07-05	2455383.75921	269.44969846	4.72370029	0.00000373	0.00000283	59.00
2010-08-15	2455424.72041	269.44958083	4.72399322	0.00000255	0.00000224	58.00
2010-09-12	2455452.55843	269.44952409	4.72418109	0.00000331	0.00000235	58.00
2010-09-24	2455464.52788	269.44952317	4.72426181	0.00000293	0.00000242	58.00
2010-09-25	2455465.50082	269.44952351	4.72426698	0.00000279	0.00000255	59.00
2010-10-19	2455489.64844	269.44952582	4.72441151	0.00000261	0.00000335	57.00


Coordinates of Barnard's Star
4.7247
4.7237
4.7227
4.7217
4.7207
Declination (degrees)
269.4518
269.4508
269.4498
269.4488
269.4478
Right Ascension (degrees)

Model Parameters

$$\alpha_{now} = \alpha_{J2000.0} + \alpha_{PM}(Y_{now} - 2000) + \pi P_{\alpha}$$
$$\delta_{now} = \delta_{J2000.0} + \delta_{PM}(Y_{now} - 2000) + \pi P_{\delta}$$

- $(\alpha, \delta)_{now}$ = current coordinates
- $(\alpha, \delta)_{J2000.0}$ = coordinates in J2000.0
- α_{PM} = annual proper motion in RA
- δ_{PM} = annual proper motion in Dec
- π = parallax of the star
- P_{α} = parallax factor in α for time Y_{now}
- P_{δ} = parallax factor in δ for time Y_{now}

Trigonometric Parallax and Proper Motion

Model Parameters (J2000.0 = JDE 2451545.0)

Initial JD:	2451545.0
Initial RA [deg]:	269.452060
Initial Dec [deg]:	4.693445
Trig Parallax ["]:	0.5454
PM in RA ["/y]:	-0.79871
PM in Dec ["/y]:	10.33777

BS

Model Time Span

Begin YYYY-MM-DD:	2009-06-01
Step Size [d]:	10
End YYYY-MM-DD:	2010-12-31

Graph Properties

Center RA [d]:	269.44974
Center Dec [d]:	4.72259
Size [d]:	.00437

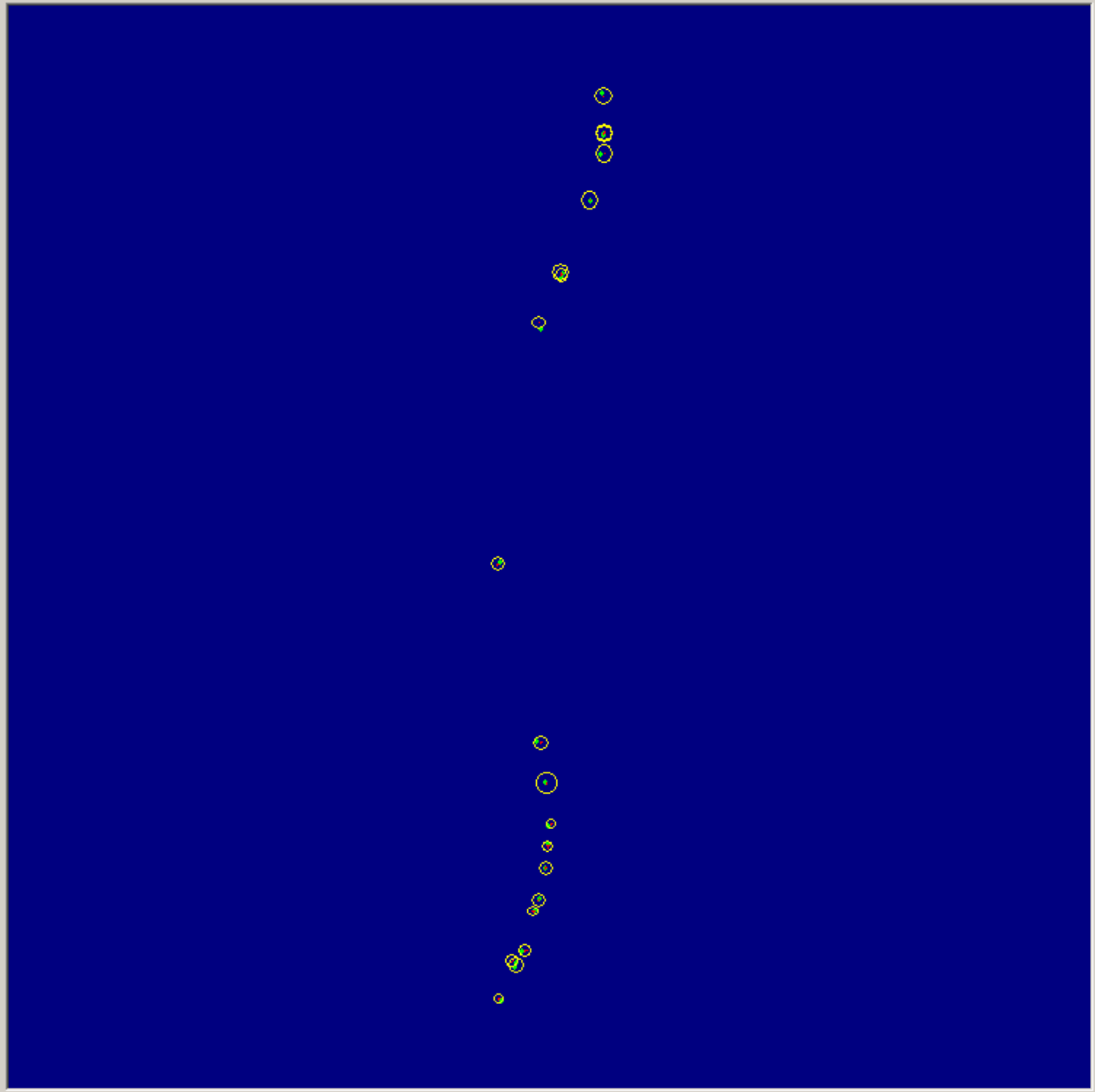
N E W S

Reinitialize | 1 arcsec

Functions

- Print Observations
- Observed Positions
- Model Observed Times
- Observed plus Model
- Model Time Span
- Display Observed minus Model
- [x] Clear Before Plotting — CLS
- [ ] Display Dates of Observations

```
RA(rms)   = 0.0000087
Dec(rms)  = 0.0000082
Residual  = 0.0000120
```



Trigonometric Parallax and Proper Motion

Model Parameters (J2000.0 = JDE 2451545.0)

Initial JD:	2451545.0
Initial RA [deg]:	269.452060
Initial Dec [deg]:	4.693445
Trig Parallax ["]:	0.5454
PM in RA ["/y]:	-0.79871
PM in Dec ["/y]:	10.33777

BS

Model Time Span

Begin YYYY-MM-DD:	2009-06-01
Step Size [d]:	10
End YYYY-MM-DD:	2010-12-31

Graph Properties

Center RA [d]:	269.44974
Center Dec [d]:	4.72259
Size [d]:	.00437

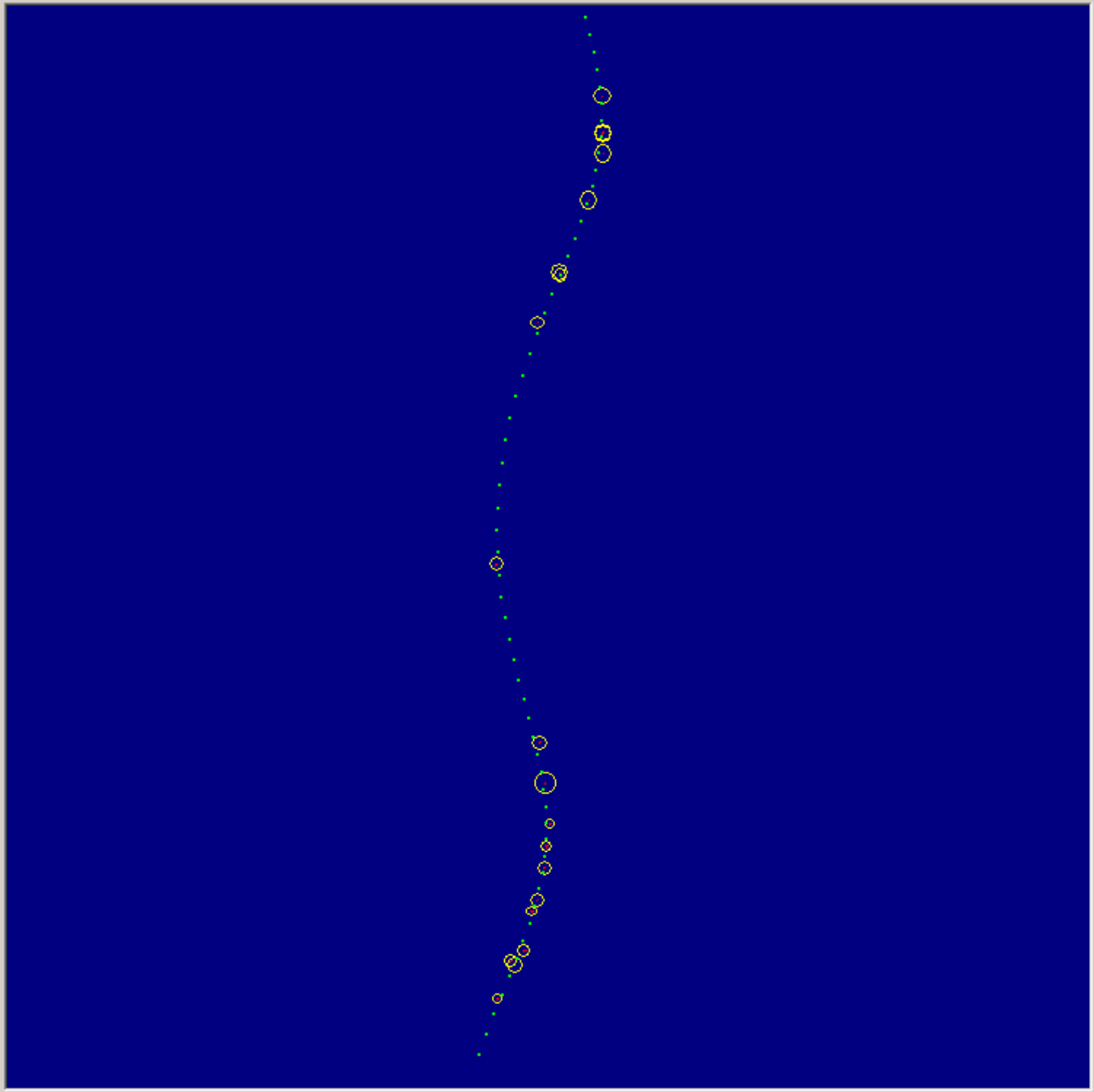
N E W S

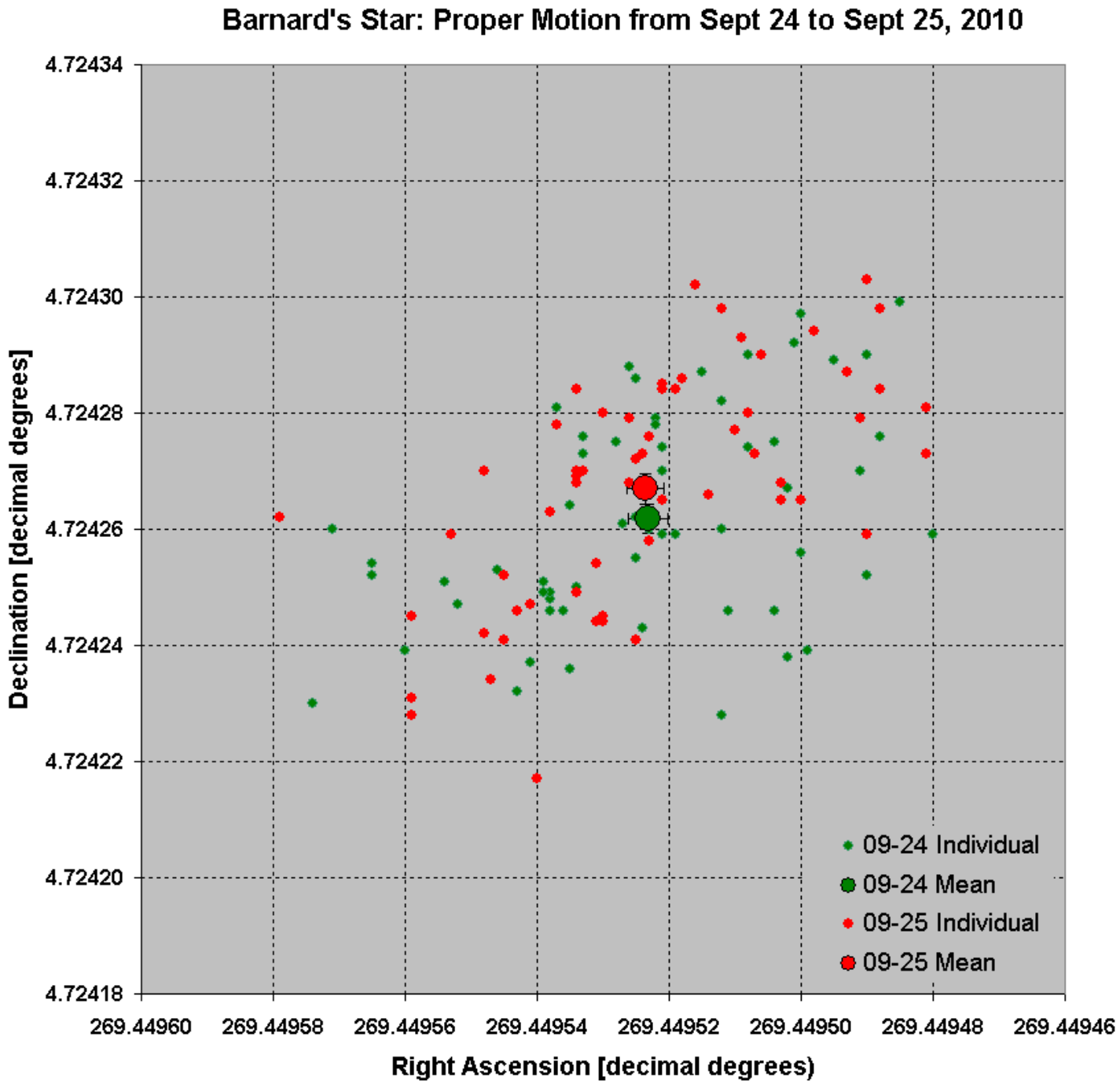
Reinitialize | 1 arcsec

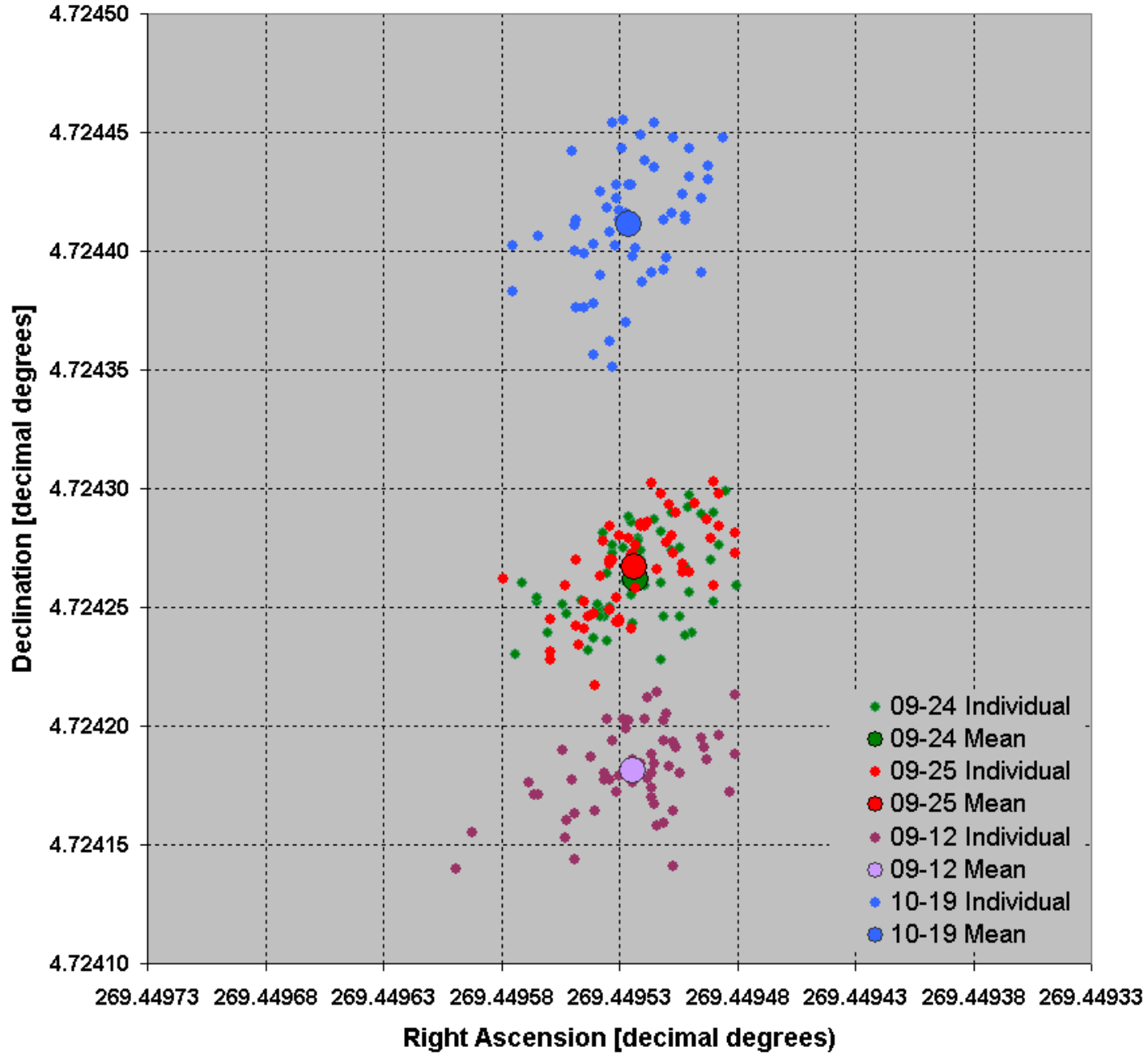
Functions

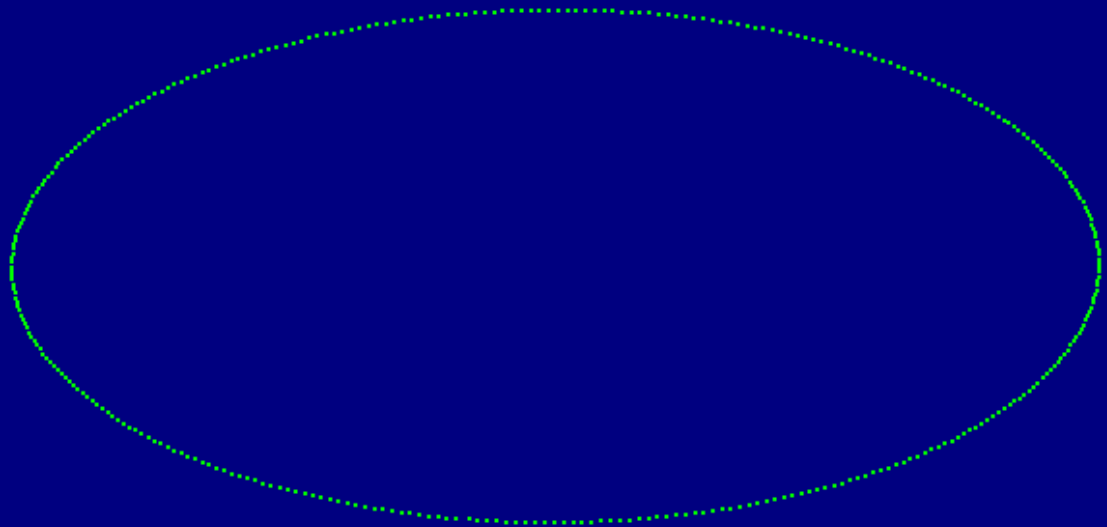
- Print Observations
- Observed Positions
- Model Observed Times
- Observed plus Model
- Model Time Span
- Display Observed minus Model
- [ ] Clear Before Plotting — CLS
- [ ] Display Dates of Observations

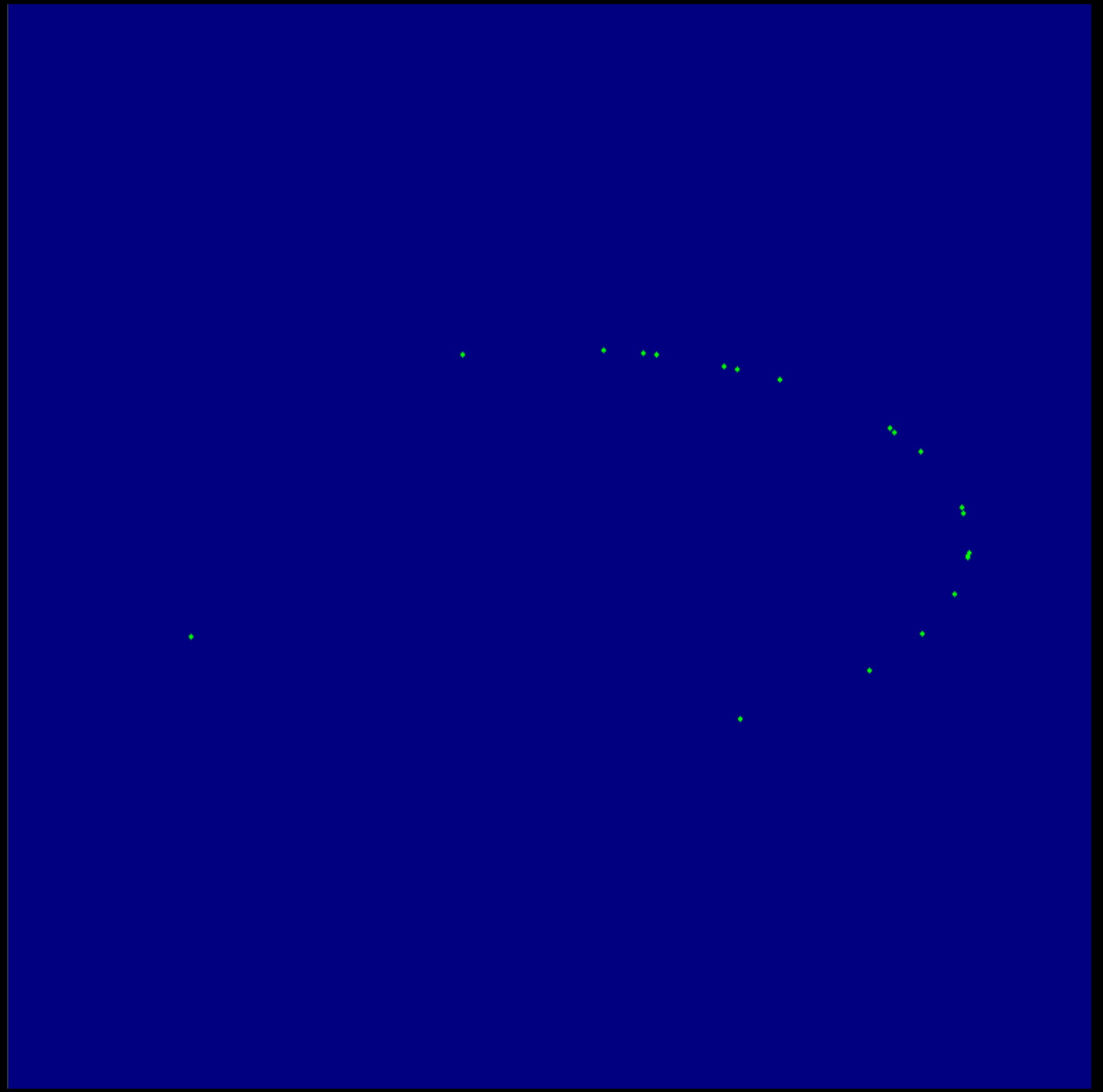
```
JD= 2455553.5
PFalpha= 0.0168
PFdelta= -0.4638
RA= 269.4496
Dec= 4.7248
```

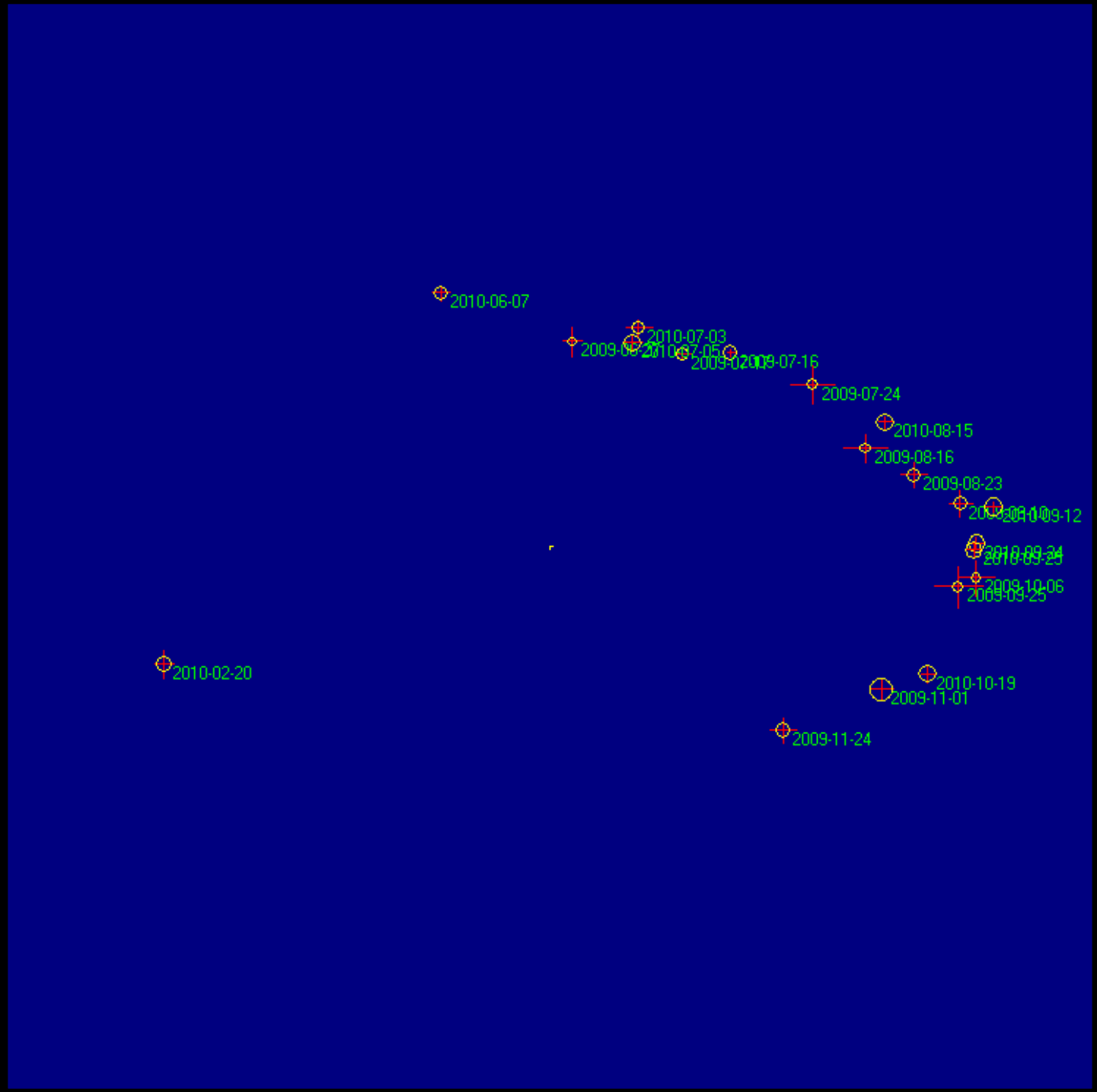


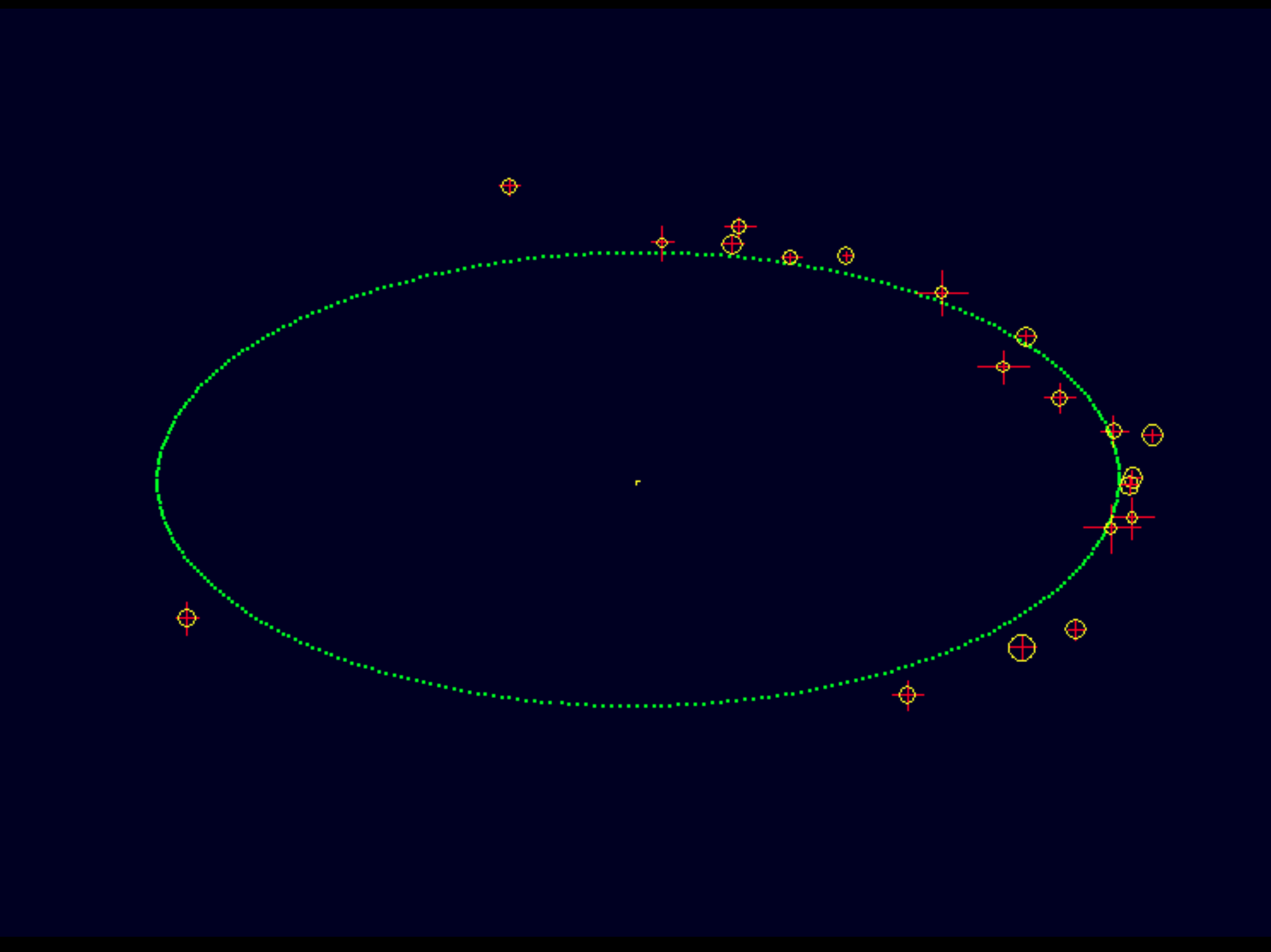

Barnard's Star: Proper Motion from Sept 24 to Sept 25, 2010
4.72434
4.72432
4.72430
4.72428
4.72426
4.72424
4.72422
4.72420
4.72418
Declination [decimal degrees]
269.44960
269.44958
269.44956
269.44954
269.44952
269.44950
269.44948
269.44946
Right Ascension [decimal degrees]
09-24 Individual
09-24 Mean
09-25 Individual
09-25 Mean


Barnard's Star: Proper Motion from Sept 12 to Oct 19, 2010
Declination [decimal degrees]
4.72450
4.72445
4.72440
4.72435
4.72430
4.72425
4.72420
4.72415
4.72410
269.44973
269.44968
269.44963
269.44958
269.44953
269.44948
269.44943
269.44938
269.44933
Right Ascension [decimal degrees)
09-24 Individual
09-24 Mean
09-25 Individual
09-25 Mean
09-12 Individual
09-12 Mean
10-19 Individual
10-19 Mean






2010-06-07
2010-07-03
2009-07-16
2009-07-24
2010-08-15
2009-08-16
2009-08-23
2009-10-06
2010-02-20
2010-10-19
2009-11-01
2009-11-24



Trigonometric Parallax and Proper Motion

Model Parameters (J2000.0 = JDE 2451545.0)

Initial JD:	2451545.0
Initial RA [deg]:	269.452060
Initial Dec [deg]:	4.693445
Trig Parallax ["]:	0.5454
PM in RA ["/y]:	-0.79871
PM in Dec ["/y]:	10.33777

BS

Model Time Span

Begin YYYY-MM-DD:	2009-06-01
Step Size [d]:	10
End YYYY-MM-DD:	2010-12-31

Graph Properties

Center RA [d]:	269.44974
Center Dec [d]:	4.72259
Size [d]:	.0001

N E W S

Reinitialize | 1 arcsec

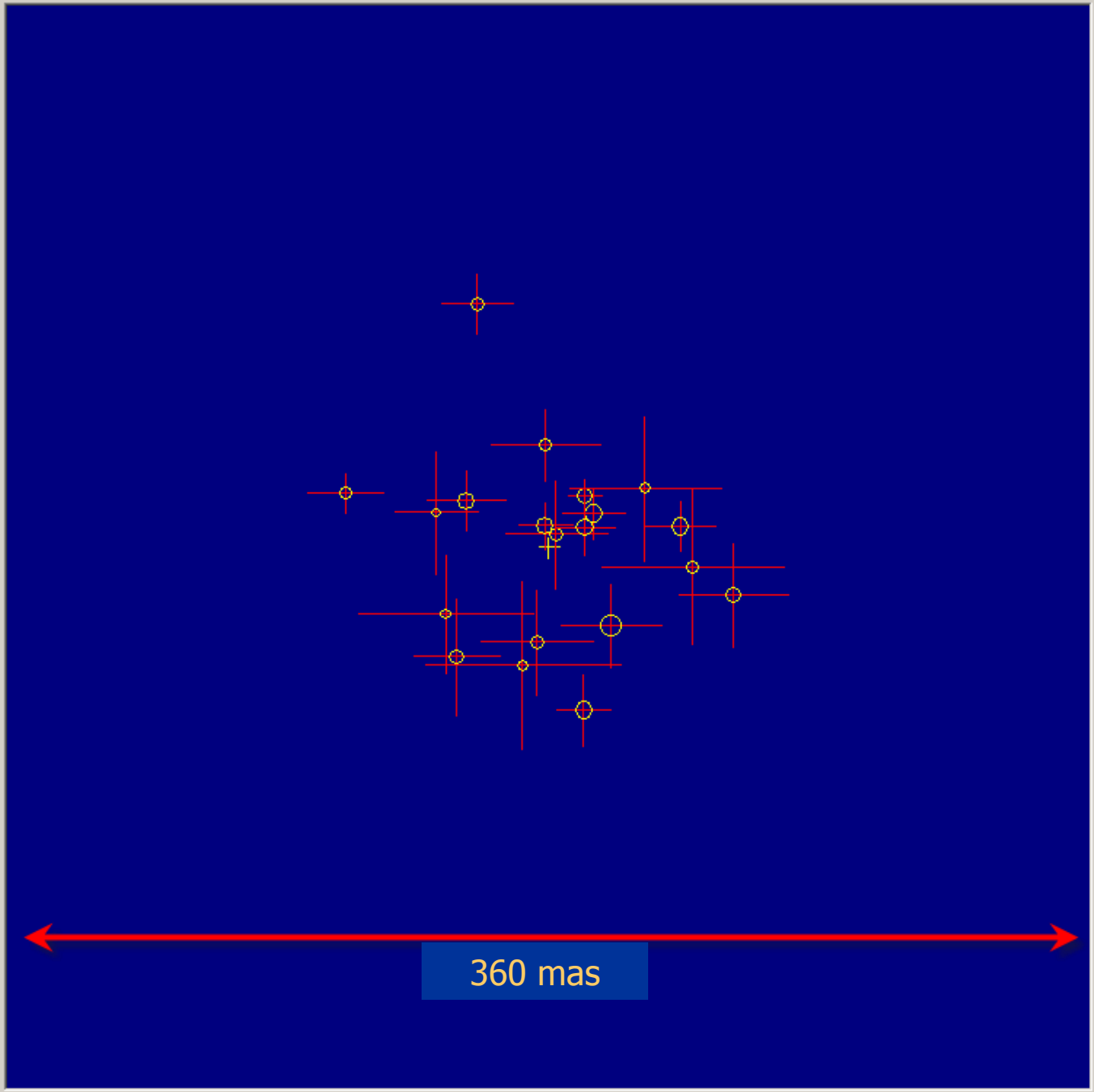
Functions

- Print Observations
- Observed Positions
- Model Observed Times
- Observed plus Model
- Model Time Span
- Display Observed minus Model

☑ Clear Before Plotting | CLS

☐ Display Dates of Observations

```
RA(rms)   = 0.0000087
Dec(rms)  = 0.0000082
Residual  = 0.0000120
```



Capability of Small-Telescope Astrometry

- Given:
  - CCD camera with 6.4 μm pixels
  - Telescope focal length ~1,000 mm
  - Large number of good reference stars
  - Maximum non-saturating exposure time
  - Observation consisting of multiple images
- Possible to:
  - Routinely achieve 0.020 arcsecond precision.
  - Sometimes achieve 0.010 arcsecond precision.

The Trigonometric Parallax and Proper Motion of Barnard's Star

Error and Precision in Small-Telescope Astrometry

Thank You!

Richard Berry